THÈSE

présentée

A L'UNIVERSITÉ PARIS-SUD

CENTRE D'ORSAY

pour obtenir

LE GRADE DE DOCTEUR ES-SCIENCES PHYSIQUES

par

Fernand HAYOT

Corrélations entre particules dans les
réactions à haute multiplicité.

Soutenue le 30 Juin 1976 devant la Commission d'Examen

Président M. R. OMNES

Examinateurs M. M. DAVIER
A. KRZYWICKI
M. LE BELLAC
A. MOREL

ORSAY
n° d'ordre : 1676

UNIVERSITE DE PARIS-SUD

CENTRE D'ORSAY

THESE

présentée

Pour obtenir

Le grade de DOCTEUR ès-sciences

PAR

Monsieur HAYOT Fernand

SUJET : Corrélation entre particules dans des réactions à haute multiplicité.

soutenue le 30 juin 1976 devant la Commission d'examen

MM. OMNES Président

LE BELLAC

DAVIER

KRZYWICKI

MOREL

Digitized by the Internet Archive
in 2026

https://archive.org/details/IA41548118_0011

La majeure partie des travaux sur lesquels cette thèse est basée ont été faits en collaboration avec A. Morel. Cette collaboration fructueuse a été et est un élément important de ma vie de physicien.

J'ai également eu la chance de collaborer avec M. Le Bellac dans le même esprit d'amitié, et dans la passion commune pour la physique.

Je suis très heureux que tous deux fassent partie du jury de thèse.

Je suis également heureux que A. Krzywicki en soit membre, lui qui a contribué de façon importante à l'étude de la structure des événements à grand nombre de particules.

Je remercie vivement M. R.Omnès d'avoir accepté de présider le jury, et M. M. Davier de bien vouloir en faire partie.

INTRODUCTION

Ce travail décrit le cheminement qui a mené à notre compréhension actuelle de quelques aspects importants concernant la structure d'événements à grand nombre de particules dans les collisions à haute énergie. Ce domaine d'étude des réactions d'interactions fortes traverse - pour le moment - une période d'accalmie. Les raisons sont essentiellement de deux sortes. L'une est générale, c'est l'effervescence suscitée parmi les physiciens expérimentateurs et théoriciens par la découverte fin 1974 des nouvelles particules très étroites à BNL et à SLAC. Tout un mouvement extrêmement vigoureux dans le domaine expérimental et théorique a suivi, centré autour de la notion de "charme", nouveau nombre quantique qui serait porté par les constituants de particules. Il s'avère que les études expérimentales les plus prometteuses sont devenues celles concernant la structure hadronique, explorée à l'aide de sondes électromagnétique (photon) ou faible (boson W) plutôt que celles des interactions entre hadrons. Cet essor expérimental - quoique les expériences en question soient extrêmement difficiles - va de pair, ou a même été précédé, par le développement de théories des champs renormalisables, bâties avec des lagrangiens avec invariance de jauge non abélienne spontanément brisée. Un modèle type, le plus remarquable pour ce qui est de la vérification de ses prédictions, est celui de Weinberg-Salam.
Ces théories fournissent un cadre d'étude, à l'intérieur duquel on connaît les questions cruciales à poser concernant la structure hadronique, ce qui permet d'en tester directement, par l'expérience, les aspects fondamentaux.
Rien d'étonnant alors que ce domaine soit devenu le plus actif en physique des particules, menant à un certain abandon de l'étude du problème très compliqué de l'interaction hadron-hadron, en particulier pour ce qui est des réactions à haute énergie avec production d'un grand nombre de particules.

Mais l'enthousiasme entourant tout ce qui touche aux nouvelles particules n'explique pas à lui seul cet abandon. L'autre raison est qu'il y a eu désenchantement avec les expériences à haute énergie, car malgré le développement de nouveaux concepts, ceux-ci ne semblent pas faire avancer clairement notre compréhension profonde des interactions fortes. De là il résulte l'incapacité des théoriciens de proposer des expériences significatives dans le domaine de l'étude de la structure des évènements à grand nombre de particules, et le découragement des expérimentateurs qui se tournent vers des expériences plus prometteuses, parce qu'elles peuvent s'interpréter dans un cadre théorique bien établi et défini. L'exception, c'est les phénomènes à grand moment transverse, en raison justement de la possibilité qu'ils semblent fournir d'étudier les collisions des constituants hadroniques et de rejoindre ainsi les préoccupations dans les autres domaines.

Pour ce qui est des réactions à haute multiplicité, le concept fondamental qui a émergé est celui d'ordre à courte portée. Ce concept s'est dégagé à partir de résultats obtenus aux ISR sur la corrélation en rapidité entre deux particules. Ces résultats montrent que deux particules voisines en rapidité sont fortement corrélées. Cet ordre à courte portée s'exprime de façon générale sur la fonction de corrélation de ces particules. Cette fonction, qui par invariance de Lorentz ne dépend que de différences en rapidités, celles du projectile, de la cible et des deux particules, devient indépendante d'une différence de rapidités dès que cette différence devient suffisamment grande. Le nombre qui caractérise cette courte portée est indépendant de l'énergie de la particule incidente, et est de l'ordre de une à deux unités en rapidité.[1] Quand les deux particules observées dans l'état final se trouvent ainsi suffisamment éloignées des bords de l'espace en rapidité, la fonction de corrélation ne dépend que de la différence de leurs rapidités. Comme toute référence à l'énergie incidente est alors absente, il s'ensuit que la valeur de la fonction de corrélation restera la même dans toute cette région centrale quand l'énergie incidente va augmenter. Ce trait remarquable et d'autres propriétés de l'ordre à courte portée sont confirmés par les résultats expérimentaux sur les corrélations.[1]

L'image physique qu'on peut donner de cet ordre à courte portée correspond à la formation dans la réaction de clusters, amas de particules, qui se désintègrent ensuite, en pions principalement.

La courte portée de la corrélation entre deux particules est alors reliée à la provenance commune de ces particules d'un même clusters. En fait cette image existait déjà avant les mesures des corrélations en rapidité. Ces dernières ont donc confirmé cette approche et l'interprétation des résultats expérimentaux en terme de production et désintégration de clusters s'est imposé. Les recherches théoriques et expérimentales se sont alors portées sur l'élucidation de la dynamique de ces clusters, surtout de leur contenu en particules. Le point important est évidemment de savoir si les clusters ne fournissent qu'un langage commode pour la description des phénomènes d'ordre à courte portée, ou si au contraire il s'agit là d'entités, peut-être nouvelles, qui renferment de l'information précieuse sur les interactions fortes

Expérimentalement, dans les réactions à haute multiplicité, ont d'abord été mesurées des valeurs moyennes, tel que le nombre moyen de particules produites et les moments de la distribution en multiplicité, ensuite des sections efficaces différentielles, inclusives et semi-inclusives à deux particules. Nous allons traiter de ces différents sujets à tour de rôle, toujours du point de vue de la physique des clusters. Nous terminerons sur quelques aspects concernant les corrélations en moments transverses, ce qui permettra de revenir à l'étude générale du concept d'ordre à courte portée.

La discussion précédente de l'ordre à courte portée est basée sur le reprint N°1, reproduit dans ce manuscrit. Dans les chapitres suivants sont décrits les éléments essentiels du contenu des autres reprints, qui sont portés en référence dans les titres de chapitres. La description suit l'ordre chronologique de la parution de ces travaux.

2)3)

1. LES CLUSTERS

Dès les débuts de l'étude des réactions avec production d'un grand nombre de particules dans l'état final, il est apparu que chaque événement à n particules (chargées) produites appartient à l'une ou l'autre des deux catégories suivantes : ou bien il est diffractif, ou bien il est non diffractif. Pour les événements diffractifs toutes les particules de l'état final proviennent de l'excitation diffractive de la cible ou du projectile et peuplent plutôt les faibles multiplicités. D'autre part les rapidités des particules produites sont concentrées aux extrémités du domaine en rapidité, limité par les rapidités de la cible et du projectile. Par contre, pour les événements non diffractifs, les particules sont distribuées en rapidité sur toute la région accessible, avec une forte production dans la région centrale, ce qui correspond à des particules lentes dans le centre de masse de la collision. Quoiqu'il soit expérimentalement difficile de séparer les événements en ces deux catégories, sans aucune ambiguité, ces modèles à deux composantes se sont trouvés vérifiés à chaque fois qu'une telle séparation a été tentée. Le problème est qu'il n'est pas aisé de définir un évènement diffractif par sa structure en rapidité. (Pour une tentative d'extraction de cette structure voir référence 1). Les événements diffractifs font intervenir le Poméron, auquel sont associés des phénomènes de corrélation à longue portée. Dans la suite de cet exposé, nous nous limitons à l'étude des événements non diffractifs (80% du total), pour lesquels a été introduite la notion de cluster. Cette introduction découle de l'observation que, alors que $f_2=0(A^1)$ pour une distribution de Poisson de particules, une production de clusters indépendants, identiques, se désintégrant chacun en un nombre fixé p de particules donne

$$f_2=(p-1)\ \langle n\rangle,$$

en accord avec les données pour lesquelles f_2 est positif quand $E_{lab}>50$ GeV/c. Le moment f_2 est positif non seulement pour les particules chargées, mais encore pour les particules négatives, ce qui exclut la production de clusters $\pi^+\pi^-$. Ceci n'est évidemment vrai que dans le cas considéré de production indépendante de clusters. De là, néanmoins, est issue l'idée que les clusters sont en fait des objets assez lourds, se désintègrant en moyenne en plus de deux particules chargées, et donc en plus de trois particules si l'on tient compte des neutres. Différents modèles ont été proposés, en particulier le modèle ρρ (étudié en détail dans l'appendice B à l'aide du formalisme de la fonction génératrice).

Ce modèle reproduit remarquablement bien les résultats expérimentaux dans la plupart des cas considérés. Il a été d'autre part généralisé au cas où les clusters se désintègrent en des paires de mésons vecteurs d'octet, la paire étant dans un état singulet de SU(3). On reproduit ainsi les sections efficaces de production de n particules chargées à haute énergie et par conséquent la courbe de KNO dans le domaine d'énergie de 50 à 300 GeV/C. On obtient donc une quasi-invariance d'échelle, laquelle n'apparaît que comme un phénomène transitoire, accidentel, conséquence dans le modèle considéré d'une compétition entre le mécanisme de production centrale et celui de la fragmentation du projectile et de la cible, qui participent tous deux aux évènements non diffractifs.

Dans toutes ces études on considère le cas de la production de clusters identiques, indépendants et neutres. Sans ces restrictions, en particulier celle d'indépendance, il y aurait trop de paramètres et de distributions ajustables dans le modèle. La conclusion de ces études a été qu'il existe de nouvelles entités dynamiques, les clusters, assez lourds, dont il est par conséquent important de mettre en lumière la structure et le rôle dans les réactions à haute multiplicité. Ces clusters doivent donner lieu à des effets dynamiques de courte portée entre particules de désintégration d'un même cluster. D'où l'intérêt présenté par les expériences inclusives et semi-inclusives sur les corrélations en rapidité entre deux particules. Ces corrélations font l'objet des deux chapitres suivants.

II. CORRELATIONS EN RAPIDITE INCLUSIVES[4)]

Dans les expériences inclusives qui nous concernent, la seule information donnée sur l'état final est celle concernant les impulsions des deux particules chargées. En particulier pour ce qui est des rapidités, on mesure la section efficace différentielle pour les particules 1 et 2, à laquelle est reliée la fonction de corrélation

$$R(y_1, y_2)$$

définie dans l'appendice A (A3).
Expérimentalement, pour les particules situées loin des extrémités de l'intervalle en rapidité, et à toute énergie, cette fonction est bien représentée par

$$R(y_1, y_2) \sim \exp - \frac{(y_1 - y_2)^2}{4\delta^2} \quad , \text{ avec } \delta \simeq 0.7.$$

Comme il est mentionné dans l'Introduction, l'indépendance par rapport à l'énergie, et la dépendance en (y_1-y_2) seulement de la fonction $R(y_1,y_2)$ sont des manifestations d'ordre à courte portée, lui-même signe d'une corrélation non négligeable à courte portée entre les particules 1 et 2. Nous soulignons que, en ce qui concerne la conservation du moment total dans la réaction, celle-ci se traduit par une corrélation qui décroît avec l'énergie, et qui se manifeste surtout entre deux particules de rapidités opposées. Une vraie corrélation de courte portée fait naturellement partie de l'image des clusters. On suppose, à nouveau, que des clusters identiques sont produits indépendamment, avec une distribution uniforme en rapidité, et que la désintégration au repos de chaque cluster est isotrope. Des corrélations sont alors créées entre deux particules provenant d'un même cluster par l'effet d'entraînement lié au mouvement du cluster. La quantité caractéristique de la taille et du mode de désintégration du cluster, mesurée par les corrélations inclusives, et qu'on peut extraire de $R(y_1,y_2)$ est le rapport (pour des particules identiques)

$$\frac{\langle p(p-1)\rangle}{\langle p\rangle} \quad ,$$

où la moyenne ici porte sur les désintégrations du cluster. Il est clair que dans un tel modèle les corrélations de courte portée entre deux particules de même charge sont proportionnelles au nombre de façons dont on peut les choisir parmi les p particules de désintégration. (Pour l'expression complète de la section efficace à deux particules dans le modèle de clusters, voir A4). Expérimentalement, on trouve que le rapport $\frac{\langle p(p-1)\rangle}{\langle p\rangle}$ est environ 2.5 pour deux particules chargées. Cela signifie que pour les clusters se désintègrant en un nombre fixé de particules on aurait p de l'ordre de trois ou quatre, et pour une distribution de Poisson on aurait $\langle p\rangle = 2.5$. Ces nombres correspondent à des clusters assez lourds.

On trouve d'autre part, expérimentalement, que dans tout le domaine d'énergie des ISR, la quantité $R(o,o)$ est pratiquement constante. On en conclut que le rapport $\frac{\langle p(p-1)\rangle}{\langle p\rangle}$ n'a, au plus, qu'une très faible variation avec l'énergie, et que, quand l'énergie incidente augmente, c'est essentiellement le nombre moyen de clusters qui croît, lequel est directement lié au nombre moyen de particules observées. On aboutit donc à une généralisation naturelle du modèle multipériphérique, avec la chaîne de clusters remplaçant la chaîne de particules.

Les corrélations inclusives en rapidité semblent donc mener à la conclusion que les clusters sont des objets assez lourds, ce qui tend évidemment à renforcer l'idée qu'il y a là un phénomène dynamique nouveau se révèlant dans la courte portée de ces corrélations.

Cependant, l'extraction du rapport $\frac{<p(p-1)>}{<p>}$ des données est entachée d'erreur, car dans les corrélations mesurées par R(lesquelles se réfèrent à la section efficace totale inélastique), il existe une composante de longue portée due à la présence des évènements diffractifs. La composante de courte portée s'ajoute donc à un fond, plat dans la région centrale, et dont l'importance est mesurée par σ_D/σ_{ND}, rapport de la section efficace totale diffractive à la non diffractive qui est de l'ordre de 20 à 30%. D'autre part, les corrélations inclusives se bâtissent à partir des corrélations semi-inclusives, lesquelles sont mesurées dans les expériences où le nombre de particules chargées dans l'état final est fixé. Là, encore, intervient une composante qui n'est pas liée à l'existence de clusters. Pour bien étudier ces derniers, il faut donc aller plus loin et regarder en détail les corrélations semi-inclusives. Celles-ci forment le sujet du prochain chapitre.

III. CORRELATIONS EN RAPIDITE SEMI-INCLUSIVES [5) 6)]

Comme il vient d'être dit, on pense à l'aide des corrélations semi-inclusives s'affranchir de l'incertitude provenant des évènements diffractifs. Pour cette raison, on considère uniquement des corrélations à grand nombre ($n>1/2<n>$) de particules chargées. Il est clair, d'autre part, qu'on ne peut pas non plus aller à des n trop grands ($n>><n>$), où l'hypothèse d'indépendance des clusters deviendrait douteuse, le nombre de clusters augmentant quand n augmente. Nous reviendrons sur ce point plus loin. L'intérêt des corrélations semi-inclusives est donc de fournir une meilleure détermination de la corrélation entre deux particules de la région centrale en rapidité. Elles permettent ainsi de vérifier la consistence du formalisme général car on peut en fait exprimer les corrélations semi-inclusives en fonction des inclusives. Comme précédemment, la quantité essentielle à extraire des données, caractéristique des propriétés de désintégration d'un cluster, est le rapport

$$r(n) = \frac{<p(p-1)>_n}{<p>_n} ,$$

lequel maintenant dépend de n. Pour $n=<n>$ ce rapport est égal à la valeur correspondante inclusive, et l'information supplémentaire contenue dans les corrélations semi-inclusives par rapport aux inclusives provient de cette dépendance en n. Pour exprimer les corrélations inclusives en fonction des semi-inclusives on utilise simplement le fait que

$$\frac{d^2\sigma}{dy_1\,dy_2} = \sum_n \frac{d^2\sigma_n}{dy_1\,dy_2}$$

De là, on déduit, utilisant les définitions de l'appendice A, que

$$C(y_1,y_2)=\sum_n P_n\, C_n(y_1,y_2)+\sum_n P_n\left[\rho_n^{(1)}(y_1)-\rho^{(1)}(y_1)\right]\left[\rho_n^{(1)}(y_2)-\rho^{(1)}(y_2)\right] \qquad (1)$$

où $P_n=\frac{\sigma_n}{\sigma_{inel}}$.

Si, pour des raisons de simplicité, on prend $\rho_n^{(1)}(y)=n/Y$, $\rho(y)=\langle n\rangle/Y$, le deuxième terme dans le membre de droite de (1) prend la forme : $\frac{1}{Y^2}\left(f_2+\langle n\rangle\right)$ qui décroit comme $\frac{1}{Y}$ avec l'énergie (Y=log s). Pour des modèles avec corrélations de courte portée où la largeur de P (n) croît comme $\langle n\rangle^{1/2}$, on a la relation

$$\sum_n P_n\, C_n = C_{\langle n\rangle} + O(1/Y) \qquad (2)$$

Tenant compte de (2), on obtient, utilisant (1),

$$C_{\langle n\rangle}(y_1,y_2)=C(y_1,y_2)-\frac{1}{Y^2}\left(f_2+\langle n\rangle\right)$$

En introduction au lieu de la distribution P_n la distribution

$$z^n P_n \Big/ \sum_n z^n P_n \ ,$$

l'expression précédente peut être généralisée à tous les n, et l'on obtient

$$C_n(y_1,y_2)=C(y_1,y_2,z)-\frac{1}{Y^2}\left(f_2(z)+n\right), \quad \text{avec } n=\langle n(z)\rangle \qquad (3)$$

Pour les modèles de clusters, on utilise l'expression donnée dans ces modèles pour $C(y_1,y_2)$ (cf. appendice A), et l'on trouve alors

$$\frac{C_n}{n}(0,y)=-\frac{1}{Y^2}\left(1+r(n)\right)+r(n)\,\frac{\exp -y^2/4\delta^2}{2\delta\sqrt{\pi}\;Y} \ . \qquad (4)$$

Remarquons que la quantité C_n/n (de même que R_n) ne dépend de n que par le rapport r(n). L'expression (4), d'autre part, est écrite pour des particules identiques. Pour deux particules non identiques le 1 n'apparaît pas dans la parenthèse de (4) et $\langle p(p-1)\rangle_n$ est remplacé par $\langle p_1 p_2\rangle_n$.

L'expression (4) de la corrélation semi-inclusive contient une prédiction remarquable du modèle des clusters, à savoir que la largeur δ est indépendante de n, parce que dans ces modèles $C(y_1,y_2)$ ne dépend pas de z. Cette prédiction est corroborée par l'expérience. Les données des ISR, pour lesquelles ont été effectuées des ajustements avec une expression telle que (4), avec un paramètre δ_n, montrent que la longueur de corrélation est pratiquement indépendante de n, et d'autre part, confirment qu'elle possède la valeur 0.7, la même que celle qui décrit les corrélations inclusives.

On constate sur (4) que la partie de courte portée est contenue dans la gaussienne $\exp -y^2/4\delta^2$.

En l'absence de corrélation ce terme est remplacé par $\frac{1}{Y}$ et l'on a $C_n/n = -\frac{1}{Y^2}$ indépendamment de y. La partie de courte portée, qui donc correspond à une gaussienne en $y_1 - y_2$, de largeur unité, se trouve sur un fond dont l'expression générale est (cf.(1))

$$\sum_n P_n \left[\rho_n^{(1)}(y_1) - \rho^{(1)}(y_1)\right]\left[\rho_n^{(1)}(y_2) - \rho^{(1)}(y_2)\right], \quad (5)$$

et qui lui aussi peut être représenté par une gaussienne plus large. Il se trouve que jusqu'à la plus haute énergie obtenue aux ISR, le fond- qui décroît néanmoins avec l'énergie par rapport au terme de corrélation - domine en fait largement la corrélation, ce qui rend l'extraction de cette dernière difficile. L'extraction - à la plus haute énergie des ISR - donne le résultat suivant[7]:

i) r(n) varie peu avec n pour n autour de<n>

ii) $r(n) \simeq 1\text{-}1.2$

Le point i) indique qu'avec une bonne approximation les clusters se désintègrent en un nombre fixe de particules. Dans ce dernier cas la relation (3) entre C_n et C devient exacte - sans correction d'énergie finie - et on peut alors déduire de ii) que $r(n) = p-1 \simeq 1$ et donc que le nombre de particules chargées dans lequel se désintègre le cluster est proche de deux. Comme d'autre part, dans la réaction $\pi^- p \to \pi^+ \pi^- X$ à 205 GeV/C, il a été observé un fort signal de ρ^o dans la région centrale ($\sim$13%), on est conduit à penser que les clusters en fait ne sont pas des objets nouveaux, manifestation de nouveaux phénomènes dynamiques, mais plutôt un ensemble de résonances mésoniques connues, telles que le ρ, le ω, etc. Il est d'ailleurs devenu habituel pour les groupes expérimentaux de montrer que leurs données peuvent être reproduites à l'aide d'une superposition de telles résonances.[7] La surestimation de la taille du cluster - obtenue à partir des corrélations inclusives - s'explique par le fait que C ne contient pas seulement les C_n, mais encore le terme croisé (5) qui dépend des distributions inclusives et semi-inclusives à une particule. Ce terme est, du point de vue corrélation, de longue portée, dans le sens qu'en prenant une particule à y_1, dans un évènement à n particules, la particule en y_2 ne se trouve pas de façon également probable dans un autre évènement à n' particules.

Quelles résonances dominent - s'il y en a - les réactions à haute multiplicité est une question difficile, car expérimentalement il est ou impossible ou très dur de les mettre en évidence directement. Il est clair, cependant, que s'il y a production de ρ^o, il y a production de ρ^+ et ρ^-, et qu'il existe des corrélations entre ces résonances, s'il y a d'autre part une compensation plus ou moins locale de la charge.

En reformulant ces observations dans le langage des clusters, on dirait que les clusters ne sont ni neutres, ni produits de façon indépendante, du moins en principe, car il est impossible de séparer une entité globale "$\rho^+ \rho^-$" de deux résonances séparées ρ^+ et ρ^- corrélées par compensation locale de la charge. D'autre part, comme cela a été déjà mentionné, quand le nombre de particules dans l'état final devient très grand, des corrélations doivent apparaître entre clusters ne serait-ce que pour des raisons cinématiques. Le rapport r(n) croît d'autant plus vite avec n que les clusters sont plus fortement corrélés, et il a été vérifié [5] que cela correspond à des f_2 négatifs pour les clusters. Tous ces aspects des clusters ont été examinés en grand détail dans un travail récent[8], dont les conclusions coïncident avec ce qui est dit plus haut.

L'image finale concernant les phénomènes à haute énergie et à grande multiplicité, devient à la fois plus simple et plus complexe : plus simple dans la mesure où il y a de fortes indications que les résonances mésoniques usuelles jouent un rôle dominant dans la production de l'état final, plus complexe car ces résonances sont chargées et soumises à des contraintes plus ou moins locales. C'est pourquoi une description en termes de clusters identiques, neutres et indépendants est une approximation grossière, valable uniquement pour la description des évènements avec un nombre de particules voisin de la valeur moyenne.

Dans tout ce qui précède, il a été question seulement de rapidités, toutes les quantités étant intégrées par rapport aux moments transverses des particules. Il semble que des moments transverses on peut extraire quelques propriétés générales de la structure des événements, où se manifeste encore un ordre à courte portée.

IV. CORRELATIONS EN MOMENTS TRANSVERSES [9][10]

Nous savons que la plupart des particules produites dans une réaction à haute énergie ont un moment transverse petit (de l'ordre de deux fois la masse du pion). (Nous ne nous occupons pas des rares phénomènes à grand moment transverse). L'une des questions à se poser est la suivante : dans quelle mesure le moment transverse d'une particule est-il affecté par celui de la résonance, ou du cluster lui-même, en particulier dans le cas où le moment transverse du cluster est grand? Il s'avère qu'expérimentalement il n'existe pas d'effet dramatique que l'on puisse associer à un mouvement transverse considérable des clusters, et donc la distribution en moment transverse des clusters est semblable de celle des particules mêmes.

L'information contenue dans la section efficace différentielle

expérimentale de deux particules est presque trop riche, car à la présence possible d'effets dynamiques se superpose par exemple l'effet Goldhaber pour des pions identiques d'impulsions voisines, et qui domine toute cette partie de l'espace de phase. Quoique des propriétés générales de modèles de clusters aient été discutées, par exemple dans le cas où la désintégration est déterminée uniquement par l'espace de phase disponible [9], il semble plus prometteur d'étudier, pour les moments transverses, l'aspect de compensation locale en rapidité. Cela a été suggéré par A. Krzywicki et D. Weingarten[11], qui ont montré comment la compensation locale en rapidité du moment transverse et l'unitarité conduisent à une borne inférieure pour la pente du Poméron échangé dans une réaction élastique à haute énergie. On peut alors penser que l'ordre à courte portée se manifestant dans les évènements à grand nombre de particules est la propriété fondamentale, à partir de laquelle on peut comprendre un grand nombre de phénomènes hadroniques, en particulier les caractéristiques de la diffusion élastique. Il s'agit donc là d'un développement important. On s'y réfère cependant toujours à la situation théorique où toutes les particules dans l'état final sont mesurées. Expérimentalement il est évidemment exclu de détecter tous les neutres. Il est donc nécessaire d'étendre le formalisme au cas d'une expérience réelle à haute énergie.[10] Les quantités à déterminer sont les fluctuations d'une fonction qui représente simplement la somme des moments transverses de particules chargées situées du même côté d'une rapidité fixée. Ces fluctuations permettent non seulement d'étudier la corrélation en fonction des rapidités, entre moments transverses de deux particules chargées, mais encore d'obtenir de l'information sur la section efficace à une particule chargée et une neutre, alors que les fluctuations mesurées concernent uniquement des particules chargées. Une expérience récente [12], à FNAL, indique que la corrélation entre moments transverses décroît fortement quand les particules s'éloignent sur une distance de une à deux unités en rapidité, distance caractéristique des effets de courte portée.

Il ressort de toutes ces considérations que c'est l'ordre à courte portée qui est le trait essentiel des événements à grand nombre de particules. C'est une notion d'autant plus importante qu'elle mène via l'unitarité à des contraintes sur les réactions à deux corps.

Nous avons développé dans cette thèse la notion de cluster, telle qu'elle a été utilisée dans le contexte des réactions à haute multiplicité. Nous avons décrit comment a évolué son interprétation physique, depuis les mesures des moments de la distribution en multiplicité jusqu'aux corrélations en rapidités et moments transverses entre deux particules, dans les cas inclusif et semi-inclusif . Au départ, il s'agissait de construire un modèle qui rende compte de la distribution presque Poissonienne des particules négatives produites dans une collision à haute énergie. Alors que pour une distribution de Poisson la dispersion est égale à la valeur moyenne du nombre de particules dans l'état final, dans les modèles avec production de clusters indépendants elle est égale à p fois la valeur moyenne, où p est le nombre de particules (supposé fixe) dans lesquelles se désintègre un cluster. Cette simple généralisation fournit un moyen de décrire les données expérimentales. Une fois les clusters introduits, les recherches théoriques et expérimentales ont porté sur l'élucidation de la nature de ces clusters et de leur rôle dynamique. L'étude des corrélations en rapidité inclusives semblait indiquer que les clusters sont des objets assez lourds, et donc peut-être des entités nouvelles apparaissant dans les réactions à haute multiplicité. Cependant, quand les résultats sur les corrélations semi-inclusives sont apparus, il s'est avéré que la taille d'un cluster, telle qu'elle avait été déduite des mesures de la corrélation inclusive, était surestimée. D'autre part, il est devenu clair que les résonances mésoniques usuelles jouent un rôle important dans la structure des corrélations dans l'état final. La description des hautes multiplicités se compliquant ainsi, on en est revenu à l'étude des propriétés générales de l'ordre à courte portée, dont on était parti et dont les clusters fournissent en fait une illustration. Cet ordre à courte portée se révèle dans la structure des événements non seulement par les corrélations en rapidité, mais encore par la compensation locale (en rapidité) de nombres quantiques, tels que la charge ou le nombre baryonique, ou du moment transverse. Par ailleurs cet ordre à courte portée se traduit via l'unitarité par des propriétés caractéristiques des réactions à deux corps, telles que le comportement en énergie du pic de diffraction[11]. Ceci est un développement important dans le cadre des programmes qui tentent de reconstruire l'objet mystérieux qu'est le Poméron à l'aide de la relation d'unitarité. les propriétés topologiques des diagrammes

décrivant les états à n particules, basées sur la dualité, y jouent un rôle également crucial. Dans tous les cas notre compréhension du Poméron passe par une meilleure compréhension des états à grand nombre de particules. Les études qui ont été faites ces dernières années concernant les réactions à haute multiplicité doivent être interprétées en ce sens. De ce point de vue, et en dehors de la mise au point de nouveaux outils théoriques, les progrès apportés dans ce domaine complexe sont indéniables.

REFERENCES

1) F. Hayot et A. Morel, Nuclear Physics B68,312 (1974) Reprint n°1 dans ce manuscrit.

2) F. Hayot et A. Morel, Nuovo Cimento letters 7,221 (1973) et 8,71(1973) Reprint n°2 dans ce manuscrit.

3) F. Hayot et A. Morel, Nuclear Physics B59,493 (1973) Reprint n°3 dans ce manuscrit.

4) F. Hayot et A. Morel, Nuclear Physics B68,323 (1974) Reprint n°4 dans ce manuscrit.
F. Hayot et A. Morel, Acta Physica Polonica B4,941 (1973)

5) F. Hayot et M. Le Bellac, Nuclear Physics B86,333 (1975) Reprint n°5 dans ce manuscrit.

6) F. Hayot, Nuovo Cimento Letters 12,676 (1975) Reprint n°6 dans ce manuscrit.

7) L. Foa, Physics Reports 22 C, 1 (1975)

8) A. Arneodo et G. Plaut, preprint Nice NTH 75/8 (1975)

9) F. Hayot, F.S Henyey et M: Le Bellac, Nuclear Physics B80,77 (1974) Reprint n°7 dans ce manuscrit.

10) F. Hayot, Physical Review D12, 2751 (1975) Reprint n°8 dans ce manuscrit.

11) A. Krzywicki et D. Weingarten, Physics Letters B50,265 (1974)

12) D. Fong et al., Physics Letters B61,99 (1976).

APPENDICE A.

Cet appendice rassemble les définitions des quantités dont il est question dans le texte. Si n désigne le nombre de particules dans l'état final, on définit <n> comme le nombre moyen de particules et le moment du second ordre comme

$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2 \qquad \text{(A1)}$$

Pour les réactions inclusives, on introduit les sections efficaces différentielles à 1 et 2 particules, et les densités correspondantes :

$$\rho^{(1)}(y) = \frac{1}{\sigma}\frac{d\sigma}{dy} \quad , \quad \rho^{(2)}(y_1, y_2) = \frac{1}{\sigma}\frac{d^2\sigma}{dy_1\, dy_2} \quad , \qquad (A2)$$

où $\sigma = \sigma_{inel.}$. Les normalisations sont :

$$\int \rho^{(1)}(y)\, dy = \langle n \rangle \, , \quad \int \rho^{(2)}(y_1, y_2)\, dy_1\, dy_2 = \langle n(n-1) \rangle$$

(pour des particules identiques)

Les corrélations sont définies par :

$$C(y_1, y_2) = \rho^{(2)}(y_1, y_2) - \rho^{(1)}(y_1)\, \rho^{(1)}(y_2)$$

ou

$$R(y_1, y_2) = C(y_1, y_2) \Big/ \rho^{(1)}(y_1)\, \rho^{(1)}(y_2) \qquad (A3)$$

Dans le modèle à clusters le plus simple (voir texte), on a

$$\rho^{(2)}(y_1, y_2) = \langle p(p-1) \rangle \bar{k}\, P^{(2)}(y_1, y_2) + \langle p \rangle^2 \bar{k}^2 P^{(1)}(y_1) P^{(1)}(y_2), \qquad (A4)$$

où $\bar{k}$ est le nombre moyen de clusters, p est le nombre de particules dans lequel se désintègre un cluster, et

$$P^{(2)}(y_1, y_2) = \frac{\exp -(y_1 - y_2)^2 / 4\delta^2}{2\delta\sqrt{\pi}\, Y} \quad , \quad P^{(1)}(y) = \frac{\langle n \rangle}{Y} \, . \qquad (A5)$$

Ici Y(=log s)désigne l'intervalle en rapidité, δ est la longueur de corrélation. (Tous les effets de bord ont été négligés).

De la même façon pour les réactions semi-inclusives, on introduit des sections efficaces à une ou deux particules, et les densités :

$$\rho_n^{(1)}(y) = \frac{1}{\sigma_n}\frac{d\sigma_n}{dy} \quad , \quad \rho_n^{(2)}(y_1,y_2) = \frac{1}{\sigma_n}\frac{d^2\sigma_n}{dy_1\,dy_2} \quad ,$$

avec les normalisations :

$$\int \rho_n^{(1)}(y)\,dy = n \quad , \quad \int \rho_n^{(2)}(y_1,y_2)\,dy_1\,dy_2 = n(n-1)$$

(pour des particules identiques)

Les corrélations sont définies par :

$$C_n(y_1,y_2) = \rho_n^{(2)}(y_1,y_2) - \rho_n^{(1)}(y_1)\,\rho_n^{(1)}(y_2)$$

$$R_n(y_1,y_2) = C_n(y_1,y_2) \Big/ \rho_n^{(1)}(y_1)\,\rho_n^{(1)}(y_2)$$

APPENDICE B.

Dans cet appendice on donne quelques formules concernant la méthode de la fonction génératrice. On les applique ensuite au cas du modèle ρρ comme un un exemple de calcul avec les clusters. Dans ce modèle chaque cluster a une probabilité 2/3 d'être dans l'état $\rho^+\rho^- \left(=\pi^+\pi^- 2\pi^0\right)$ et 1/3 d'être dans l'état $\rho^0\rho^0 \left(= 2\pi^+ 2\,\pi^-\right)$.

Le point de départ du formalisme est l'observation que la fonction génératrice pour la distribution en multiplicité

$$G(z) = \sum_n P_n z^n$$

est égale à

$$G(z) = S(\lambda(z)) \qquad (B1)$$

où S (z) est la fonction génératrice pour la production de clusters, λ(z) celle pour la désintégration d'un cluster. A partir de G(z) il est facile de calculer les différents moments, avec leur dépendance en n, par exemple :

$$f_1(z) = \bar{n}(z) = z \frac{\partial}{\partial z} \log G(z)$$

$$f_2(z) = z^2 \frac{\partial^2}{\partial z^2} \log G(z), \text{ etc.}$$

Pour $z = 1$, $\bar{n}(1) = \langle n \rangle$, $f_2(1) = f_2$.

Modèle ρρ.

Les clusters sont produits indépendamment, donc dans

$$S(z) = \sum_n z^n Q(n)$$

Q(n) est donné par une distribution de Poisson, i.e

$$Q(n) = e^{-\bar{k}} \bar{k}^n / n!$$

Par conséquent

$$S(z) = e^{\bar{k}(z-1)} \qquad (B2)$$

, où $\bar{k}$ est le nombre moyen de clusters.

Pour les particules négatives dans lesquelles se désintègre le cluster ρρ, on a :

$$\lambda(z) = \sum z^P W(P) = \frac{2}{3} z + 1/3\, z^2 \qquad (B3)$$

En appliquant (B1) on trouve alors pour la fonction génératrice G :

$$G(z) = e^{\bar{k}(\lambda(z)-1)}$$

avec $\lambda(z)$ donné par (B3).

1. Relation de récurrence sur les P(n).

Comme $G(z) = \sum z^n P(n)$ par définition, on a

$$P(n) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} G(z) \Big|_{z=0}$$

$$= \frac{\bar{k}}{n!} \frac{\partial^{n-1}}{\partial z^{n-1}} \lambda'(z)\, G(z) \Big|_{z=0}$$

$$= \frac{\bar{k}}{n!} \left[\lambda'(z) \frac{\partial^{n-1}}{\partial z^{n-1}} G(z) + (n-1)\, \lambda''(z) \frac{\partial^{n-2}}{\partial z^{n-2}} G(z) \right] \Big|_{z=0}$$

Tous les autres termes sont nuls puisque $\lambda(z)$ est un polynôme du deuxième degré.

Par conséquent, dans le modèle ρρ :

$$P(n) = \frac{\bar{k}}{n!} \frac{2}{3} \left[(n-1)!\, P(n-1) + (n-1)!\, P(n-2) \right]$$

On obtient la relation de récurrence :

$$P(n) = \frac{2}{3} \frac{\bar{k}}{n} \left[P(n-1) + P(n-2) \right], \qquad (B4)$$

avec $P(0) = G(0) = e^{-\bar{k}}$

et $P(1) = \frac{2}{3} \bar{k}\, e^{-\bar{k}}$

Le nombre moyen de clusters $\bar{k}$ est relié au nombre moyen de particules négatives par

$$\bar{n}_- = z \frac{\partial}{\partial z} \log G(z) \Big|_{z=1} = \bar{k}\, \lambda'(1) = \frac{4}{3} \bar{k}\,.$$

2 Calcul de $\bar{n}_o(n-)$

Une quantité importante dans ces modèles est $\bar{n}_o(n-)$, le nombre moyen de particules neutres pour un nombre donné de particules négatives. On introduit alors la fonction génératrice $G(z,z_o)$,

où
$$G(z,z_o) = \sum_{n,n_o} z^n z_o^{n_o} P(n,n_o)$$

Par définition
$$\bar{n}_o(n_-) = \frac{\sum_{n_o} n_o P(n,n_o)}{\sum_{n_o} P(n,n_o)}$$

et par conséquent, en terme de $G(z,z_o)$
$$\bar{n}_o(n_-) = \frac{\frac{1}{n!}\frac{\partial^n}{\partial z^n}\frac{\partial}{\partial z_o} G(z,z_o)\Big|_{\substack{z_o=1\\z=0}}}{\frac{1}{n!}\frac{\partial^n}{\partial z^n} G(z,z_o)\Big|_{\substack{z_o=1\\z=0}}}$$

Dans le modèle ρρ :
$$G(z,z_o) = e^{\bar{k}(\lambda(z,z_o)-1)}$$
$$et \quad \lambda(z,z_o) = \frac{2}{3} z z_o^2 + 1/3 z^2,$$

pour les particules neutres et négatives.

De la même façon que précèdemment on a
$$\frac{1}{n!}\frac{\partial^n}{\partial z^n} G(z,z_o) = \frac{\bar{k}}{n!}\left[\lambda'_z(z,z_o)\frac{\partial^{n-1}}{\partial z^{n-1}} G(z,z_o) + (n-1)\lambda''_z(z,z_o)\frac{\partial^{n-2}}{\partial z^{n-2}} G(z,z_o)\right].$$

D'autre part
$$\lambda'_z(z,z_o) = \frac{2}{3} z_o^2 + \frac{2}{3} z \quad , \quad \lambda'_{z_o}(z,z_o) = \frac{4}{3} z z_o$$
$$\lambda''_z(z,z_o) = \frac{2}{3} .$$

Donc :
$$\frac{1}{n!}\frac{\partial}{\partial z_o}\frac{\partial^n}{\partial z^n} G(z,z_o) = \frac{\bar{k}}{n!}\left[\frac{4}{3} z_o \frac{\partial^{n-1}}{\partial z^{n-1}} G(z,z_o) + \bar{k}\lambda'_z \frac{\partial^{n-1}}{\partial z^{n-1}}\lambda'_{z_o} G(z,z_o)\right.$$
$$\left. + \bar{k}(n-1)\lambda''_z \frac{\partial^{n-2}}{\partial z^{n-2}}\lambda'_{z_o} G(z,z_o)\right]$$

Ainsi, à z=0, z_o=1, on trouve

$$\bar{n}_0(n_-) = \frac{1}{P(n)} \frac{4}{3} \frac{\bar{k}}{n} \left[P(n-1) + \frac{2}{3} \bar{k} P(n-2) + \frac{2}{3} \bar{k} P(n-3) \right]$$

En tenant compte de la relation (B4), ceci se met sous forme simple :

$$\bar{n}_0(n_-) = \bar{n}_- \frac{P(n_- - 1)}{P(n_-)} \quad . \qquad (B5)$$

Cette expression reproduit bien les données expérimentales tant que n- n'est pas trop grand [2].

REPRINT N°1

TEST OF THE SRO HYPOTHESIS FOR TWO CHARGED PARTICLE CORRELATIONS AND RELATIVE SIZE OF THE DIFFRACTIVE AND NON-DIFFRACTIVE COMPONENTS

REPRINT N°1

Nuclear Physics B68 (1974) 312–322. North-Holland Publishing Company

TEST OF THE SRO HYPOTHESIS FOR TWO CHARGED PARTICLE CORRELATIONS AND RELATIVE SIZE OF THE DIFFRACTIVE AND NON-DIFFRACTIVE COMPONENTS

F. HAYOT and A. MOREL
Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay, BP 2, 91190 Gif-sur-Yvette, France

Received 16 August 1973

Abstract: It is shown that the SRO hypothesis is well satisfied by the recent ISR data on two charged particle correlations at different energies. Moreover, within the framework of the two component model, we determine from the same data the relative amount of diffractive and non-diffractive contributions as a function of the cosmic ray variable η. This leads to a consistent description of the long range correlation.

1. Introduction

There now exists a large set of good data on two charged particle rapidity correlations at ISR energies [1]. In this paper, we first check the validity of the short range order hypothesis [2, 3] in a model independent way, and secondly, in the framework of the two component model [4], we determine the relative importance of the diffractive contribution compared to the non diffractive one as a function of the cosmic ray variable $\eta = -\log \operatorname{tg} \frac{1}{2}\theta$.

2. Test of the short-range order hypothesis (SRO)

We shall formulate the SRO hypothesis in the way it is done in ref. [3], namely under the forms SRO1 and SRO2:

SRO1: any n-particle inclusive cross section, averaged over all transverse momenta, and considered as a function of $n+1$ rapidity differences $y_{i+1}-y_i$ (Lorentz invariance), depends no longer on any particular variable $y_{k+1}-y_k$ as soon as the absolute value of this variable is larger than some fixed (energy independent) number L_k. This statement is an elegant way to express the scaling property.

SRO2: For $y_{k+1}-y_k \geq L_k$, the cross section *factorizes* into two functions, one of them depending on rapidities $y_i > y_{k+1}$ only, the other one on rapidities $y_j < y_k$ only. SRO2 clearly implies SRO1.

We consider the two particle rapidity correlation function

$$R(y_1, y_2) = \frac{\sigma_{in}\, d^2\sigma/dy_1\, dy_2}{\dfrac{d\sigma}{dy_1}\dfrac{d\sigma}{dy_2}} - 1. \tag{1}$$

In the center of mass, at a given energy $\sqrt{s}$, we have

$$R(y_1, y_2) = F(y_1 + \tfrac{1}{2}\log s,\ y_2 - y_1,\ \tfrac{1}{2}\log s - y_2). \tag{2}$$

We now test SRO1 with respect to the data for various values of the three variables y_{10}, y_{21} and y_{02} in $F(y_{10}, y_{21}, y_{02})$ (relation (2)).

2.1. y_{10} and $y_{02} > L_0$, R depends only on y_{21}

We use the highest energy data ($\sqrt{s} = 63$ GeV) in order to allow a sufficiently large rapidity interval*. The result is presented in fig. 1 where $R(\eta_1, \eta_2)$ is plotted** *versus* $\eta_2 - \eta_1$ for $\eta_1 = 0$, -0.87 and -1.77. All points clearly fall on the same curve for

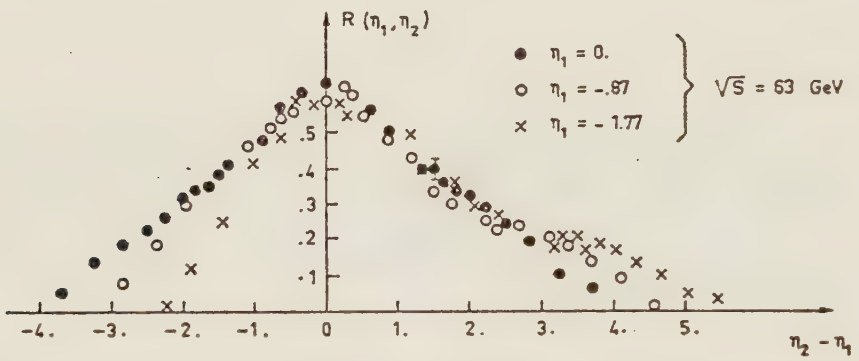

Fig. 1. Check of SRO1 at $\sqrt{s} = 63$ GeV on $R(\eta_1, \eta_2)$ as a function of $\eta_2 - \eta_1$ for different η_1 (see subsect. 2.1).

$0.5 \lesssim \eta_2 - \eta_1 \lesssim 2.5$. At $\eta_1 = -1.77$, the points $\eta_2 - \eta_1 < -0.5$ now differ from the corresponding ones at lower values of $|\eta_1|$. This means that $|\eta| \gtrsim 2$ corresponds to values of the rapidity which no longer verify the required condition.

* The variable which is used experimentally is $\eta = -\log \operatorname{tg} \frac{1}{2}\theta$ which is equal to y for a small mass (or large transverse momentum) particle only. Instead of transforming our results from one variable to the other, which would require assumptions about the $p_\perp$ distributions, we prefer to compare directly the data to the SRO predictions written in terms of η.

** Such a plot has already been presented at the Batavia Conference (1972) by the Pisa-Stony Brook group [5].

2.2. $y_{21} > L_1$ and $y_{02} > L_0$, R is independent of y_2 (plateau)

Such a plateau is clearly present in the $\eta_1 = -1.77$ and $\eta_1 = -2.8$ data (see fig. 2). It occurs for $1 \lesssim \eta_2 \lesssim 2$. This provides us with the estimate $L_1 \simeq 3$, and confirms that $|\eta| \gtrsim 2$ does not verify $y_{02} > L_0$. The same kind of effect shows up when we compare the η_2 dependence at large η_2 for various η_1 not too far from the center (now $y_{21} > L_1$ and $y_{10} > L_0$). Fig. 3 demonstrates again that the values $\eta_1 = 0$ and -0.87 verify $y_{10} \gtrsim L_0$, whereas for $\eta_1 = -1.77$ one probably has $y_{10} \sim L_0$.

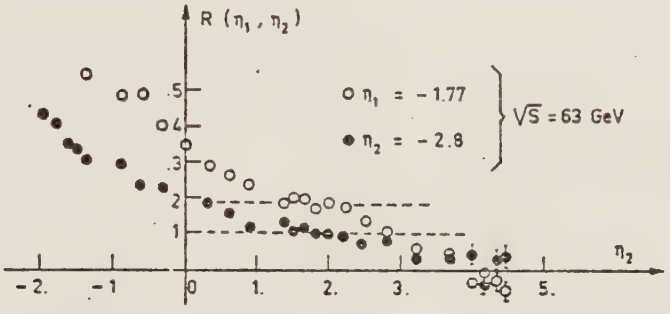

Fig. 2. The plateau of $R(\eta_1, \eta_2)$ in η_2 at $\sqrt{s} = 63$ GeV for $\eta_1 = -1.77$ and -2.8 (see subsect. 2.2).

We now turn to the study of the energy dependence of R, as predicted by the SRO1 hypothesis and formula (2). Noticing that between $\sqrt{s} = 23$ and $\sqrt{s} = 63$ GeV, the maximum c.m. rapidity is shifted by one unit, we may consider two cases of interest.

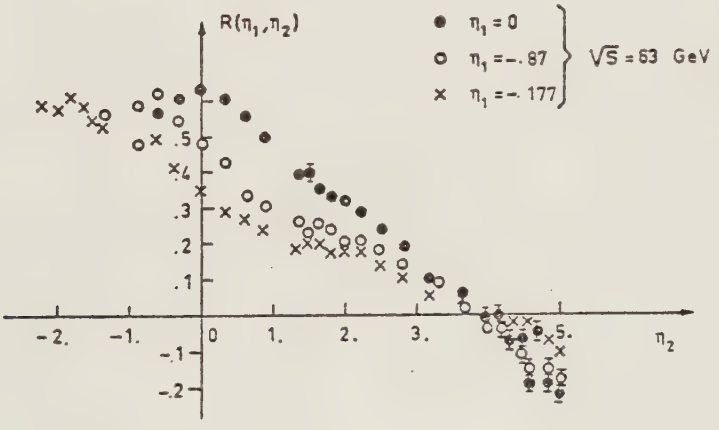

Fig. 3. Check of the independence of $R(\eta_1, \eta_2)$ at $\sqrt{s} = 63$ GeV with respect to η_1 at large η_2 (see subsect. 2.2).

2.3. $y_{21} > L_1$

Then

$$R_{23}(y_1, y_2) = R_{63}(y_1 - 1, y_2 + 1). \tag{3}$$

Here the subscript in R indicates the total c.m. energy. Again identifying η and y, we are able to make the comparison with the data, since we dispose of values of η_1 which differ approximately by one unit. Figs. 4a and b show $R_{23}(-0.87, \eta_2)$ compared

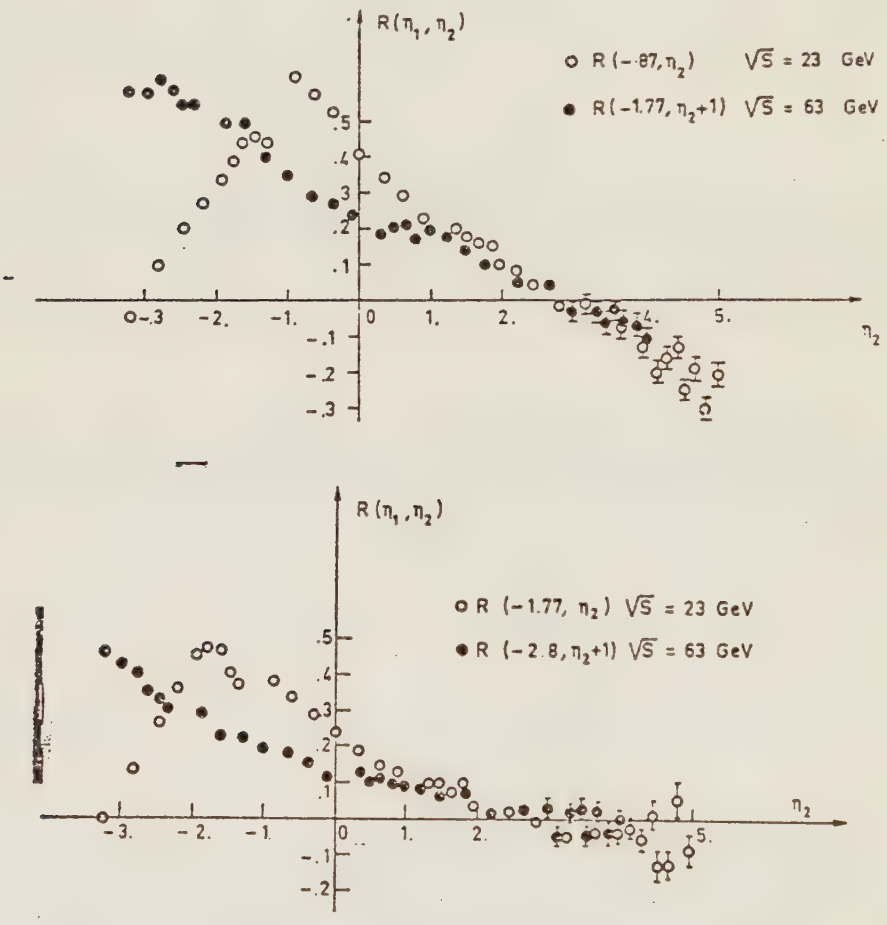

Fig. 4. Comparison of $R(\eta_1, \eta_2)$ at $\sqrt{s} = 23$ GeV with $R(\eta_1 - 1, \eta_2 + 1)$ at $\sqrt{s} = 63$ GeV (see subsect. 2.3). (a) $\eta_1 = -0.87$, (b) $\eta_1 = -1.77$.

to $R_{63}(-1.77, \eta_2 + 1)$ and $R_{23}(-1.77, \eta_2)$ compared to $R_{63}(-2.8, \eta_2 + 1)$ respectively. The agreement is quite good in both cases as soon as η_2 is larger that about two. Note that this again leads us to a value of L_1 of the order of three.

2.4. $\frac{1}{2}\log s - y_1$ *and* $\frac{1}{2}\log s - y_2 > L_0$ *at* $\sqrt{s} = 23$ *(and a fortiori at* $\sqrt{s} = 63$*)*

Then

$$R_{23}(y_1, y_2) = R_{63}(y_1 - 1, y_2 - 1). \tag{4}$$

Figs. 5a and b shows the comparison between the two quantities for $\eta_1 = -0.87$ and $\eta_1 = -1.77$ respectively. The agreement is still very good, and even excellent at the left

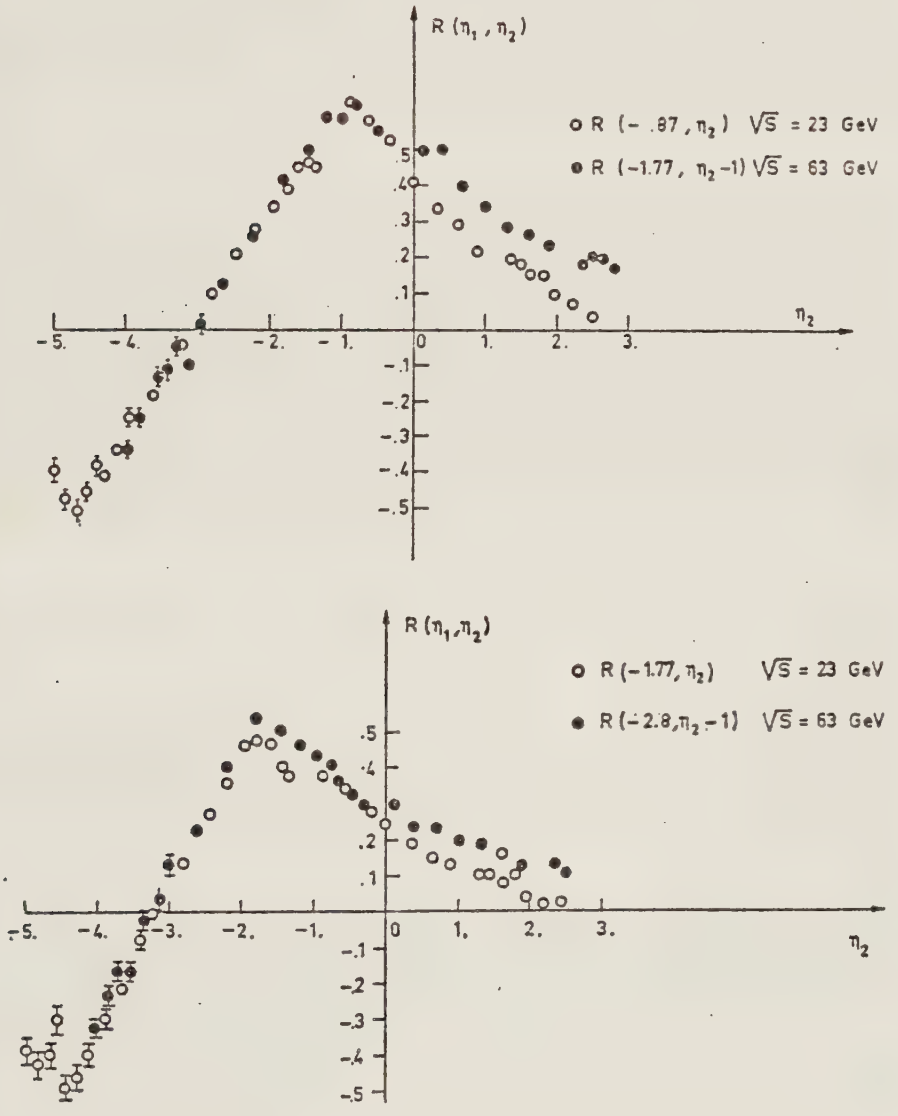

Fig. 5. Comparison of $R(\eta_1, \eta_2)$ at $\sqrt{s} = 23$ GeV with $R(\eta_1 - 1, \eta_2 - 1)$ at $\sqrt{s} = 63$ GeV (see subsect. 2.4). (a) $\eta_1 = -0.87$, (b) $\eta_1 = -1.77$.

of $\eta_1 = \eta_2$ because this condition is certainly well verified. The fact that on the right, the discrepancy shows up at $\eta_2 = 0$, which now corresponds strictly to $y_2 = 0$ indicates that L_0 is of the order of $\log 23 = 3.1$, i.e. of the same order of magnitude as L_1.

We conclude this section by stating that we have shown that the SRO1 assumption is in very good agreement with the Pisa-Stony Brook data, provided the characteristic lengths L_0 and L_1 have the common value of about three in rapidity. As far as the SRO2 assumption is concerned, we just point out that checking it consists in verifying that $1 + R(y_1, y_2)$ is of the form $A(y_1)\, A(y_2)$, when $|y_1 - y_2|$ is larger than L_1. Because of the fact that in this rapidity domain R is rather small, it turns out to be difficult to draw a definite conclusion about the validity of this stronger assumption. We shall come back to this point within the framework of the two component model which we shall now discuss and compare to the data.

3. The relative size of the diffractive and non-diffractive components as a function of η

In the two component model, one considers that any n-particle inclusive cross section is the sum of two terms, one being pure diffraction and the other one being some kind of pionization. The existence of pure diffractive events, which correspond to practically no central production in rapidity, has been recently experimentally proven [6]: practically all particles associated with a $x \simeq 1$ proton are emitted inside a very small cone in the opposite direction. On the contrary, pionization is assumed to be responsible for central production. The corresponding inclusive one particle cross sections are defined as

$$\frac{1}{\sigma_d}\frac{d\sigma^d}{d\eta} = D(\eta),$$
$$\frac{1}{\sigma_{nd}}\frac{d\sigma^{nd}}{d\eta} = \dot{\pi}(\eta), \tag{5}$$

where σ_d is the diffractive total cross section and $\sigma_{nd} = \sigma_{in} - \sigma_d$, the remaining inelastic cross section. Neglecting from now on double diffraction, we may write moreover

$$D(\eta) = g(\eta) + G(\eta). \tag{6}$$

Here $g(\eta)$ is the single proton spectrum, and $G(\eta)$ the associated spectrum of the diffractively produced particles. g and G are symmetric in η, and presumably zero or very small (especially g) for small η. In what follows, we shall take $D(0) = 0$.

In a two particle inclusive cross section, we know that there exists a short range correlation effect, which favours small $|y_1 - y_2|$ and is particularly strong in the central region. In this rapidity region, where pionization is assumed dominant, the long range correlation due to the two component mechanism has already been studied [7], and also the

short range one in terms of independent, isotropic cluster production [8]. We shall now concentrate upon the region $|y_1 - y_2| > L_1$ ($L_1 \sim 3$, as discussed in sect. 2), and extract from the Pisa-Stony Brook data the ratio D/π as a function of η without making any particular model for diffraction and pionization. For a two particle cross section, we have three possible processes, namely pionization, diffraction with the proton of one beam being elastic, and diffraction with the proton of the other beam being elastic. For each of these processes, we assume not only factorization (SRO2), but also, what Krzywicki [2] calls strong SRO, namely

$$\frac{1}{\sigma_{nd}} \frac{d\sigma^{nd}}{d\eta_1\, d\eta_2} = \pi(\eta_1)\, \pi(\eta_2) \qquad \text{for} \qquad |\eta_1 - \eta_2| \quad \text{large}, \tag{7}$$

$$\frac{1}{\sigma_d} \frac{d\sigma^{d}}{d\eta_1\, d\eta_2} = (g(\eta_1)\, G(\eta_2) + g(\eta_2)\, G(\eta_1)), \tag{8}$$

for η_1 and η_2 of opposite signs, and $|\eta_1 - \eta_2|$ large enough. This means that in this particular configuration, the two particle cross section *for each of the three processes* factorizes into the product of *the two corresponding one particle cross sections*. Finally, if η_1 and η_2 are of the same sign, and if $|\eta_1 - \eta_2|$ is large, then one of the $|\eta|$'s, $|\eta_2|$ say, is large. Then we approach the region where the proton spectrum dominates and thus the two particle diffractive cross section is small. From eqs. (7) and (8) we get*, for $|\eta_1 - \eta_2| \gtrsim 3$

$$R(\eta_1, \eta_2) = \frac{\sigma_{in}[\sigma_d(g_1 G_2 + g_2 G_1) + \sigma_{nd}\pi_1\pi_2]}{(\sigma_d D_1 + \sigma_{nd}\pi_1)(\sigma_d D_2 + \sigma_{nd}\pi_2)} - 1, \tag{9}$$

or

$$R(\eta_1, \eta_2) = \alpha \frac{1 - \alpha Z_1 Z_2 - Z_1 - Z_2 + (1+\alpha)(g_1 G_2 + g_2 G_1)/\pi_1\pi_2}{(1+\alpha Z_1)(1+\alpha Z_2)}, \tag{10}$$

where we have set $\alpha = \sigma_d/\sigma_{nd}$ and $Z = D/\pi$. The last term, of the form gG, is absent whenever η_1 and η_2 are in the same hemisphere.

We now take the data for $R(0, \eta_2)$ at $\sqrt{s} = 63$ GeV, and since by assumption $D(0) = 0$, we obtain for $|\eta_2| \gtrsim 3$

$$R(0, \eta_2) = \alpha \frac{1 - Z_2}{1 + \alpha Z_2}. \tag{11}$$

We thus determine Z_2 in this η_2 region, for a given α. In order to obtain Z_2 for η_2 below three, we first consider $R(-1.77, 1.77)$. Our eq. (10), with $g_2 \ll G_2$ (the proton spectrum is certainly concentrated at large values of η), takes the form

$$R(-1.77, 1.77) = \alpha \frac{1 - 2Z - \alpha Z^2}{(1+\alpha Z)^2}, \tag{12}$$

where $Z = Z(-1.77) = Z_1 = Z_2$.

* We use the short hand notation $A(\eta_{1,2}) = A_{1,2}$.

At this point we want to make two comments:

(i) From eq. (12), we get $\alpha > R(-1.77, 1.77) \simeq 0.18$;

(ii) $R(-1.77, \eta_2)$ shows a plateau for $1 \lesssim \eta_2 \lesssim 2$, as explained in the first section. If α were large ($\alpha \sim 0.5$), eq. (12) would give Z very different from zero, and thus presumably rapidly varying, thus spoiling the plateau.

$Z(-0.87)$ can be deduced from $R(-0.87, 1.77)$ (eq. (12)) with $g = 0$, and once this is done we use $R(-0.87, \eta_2)$ as a cross-check for $Z(\eta_2)$, when $3 \leqslant \eta_2 \leqslant 4$ and for determining it when $2 \leqslant \eta \leqslant 3$. The result for αZ is shown on fig. 6 for $\alpha = 0.3$, a value of σ_d/σ_{nd} which is consistent with a good overall fit of $R(\eta_1, \eta_2)$ when $|\eta_1 - \eta_2|$ is large

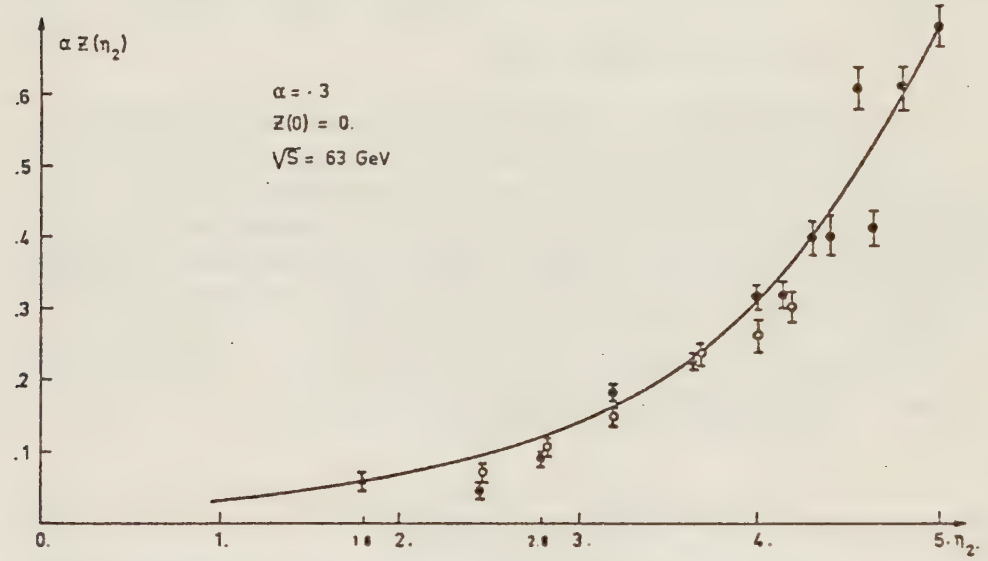

Fig. 6. Plot of $\alpha D/\pi$ *versus* η_2 at $\sqrt{s} = 63$ GeV for $\alpha = 0.3$. The black dots are determined from $R(0, \eta_2)$, the open circles from $R(-0.87, \eta_2)$ and the point at $\eta_2 = 1.77$ from $R(-1.77, 1.77)$, as explained in the text. The curve is hand-drawn and is used to obtain the predictions shown in fig. 7.

(see below), but cannot be considered as required by the data. We consider that the data favour a value α between 0.25 and 0.35. Fig. 6 also shows a smooth curve drawn by hand to guide the eye and for further use in a fit to all data. The error bars correspond only to the quoted experimental errors on R and not to the uncertainty on α. This uncertainty shows up especially for the lower η_2 values, and consequently our point $Z(-0.87)$ should not be trusted too much.

With this curve for $Z(\eta)$, we finally come back to eq. (10) to compute $R(\eta_1, \eta_2)$ as a function of η_2, for fixed values of $\eta_1 \leqslant 0$. As explained above, for $\eta_2 < 0$, we omit the gG terms in eq. (10). On the right, $\eta_2 > 0$, we make two extreme assumptions about these terms. One is

$$g_1 G_2 + g_2 G_1 = Z_1 Z_2, \tag{13}$$

which corresponds to either $D_1 = g_1$ and $D_2 = G_2$ or *vice versa*, and the other one is

$$g_1 G_2 + g_2 G_1 = 0, \tag{14}$$

which corresponds to D_1 and D_2 being both of type g or of type G. Our results are shown in figs. 7a, b, c and d, for $\eta_1 = 0$, -0.87, -1.77 and -2.8 respectively. The upper curves correspond to eq. (13), the lower ones, eq. (14), are symmetrical in η_2 since the gG terms are missing on both sides. Apart from the short range part of the correlations which has to be added to our curves [8], we clearly get a very good fit to the data. The effect of the gG terms is especially visible at $\eta_1 = -2.8$, where diffraction is important. Note that it is very likely that the actual curve should lie near the lower one at small

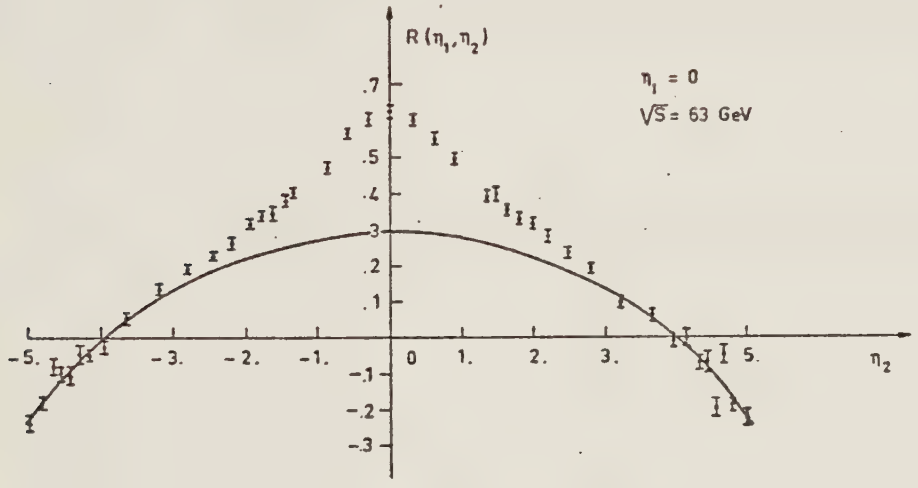

Fig. 7a.

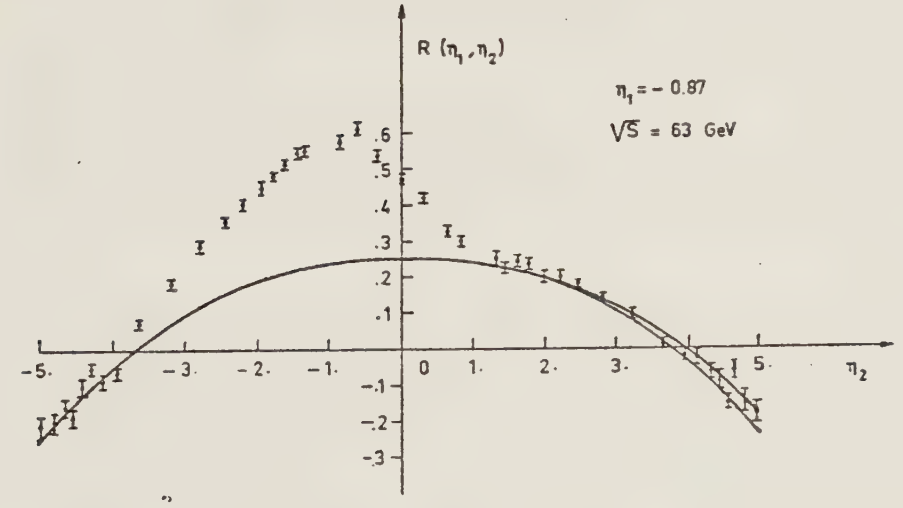

Fig. 7b.

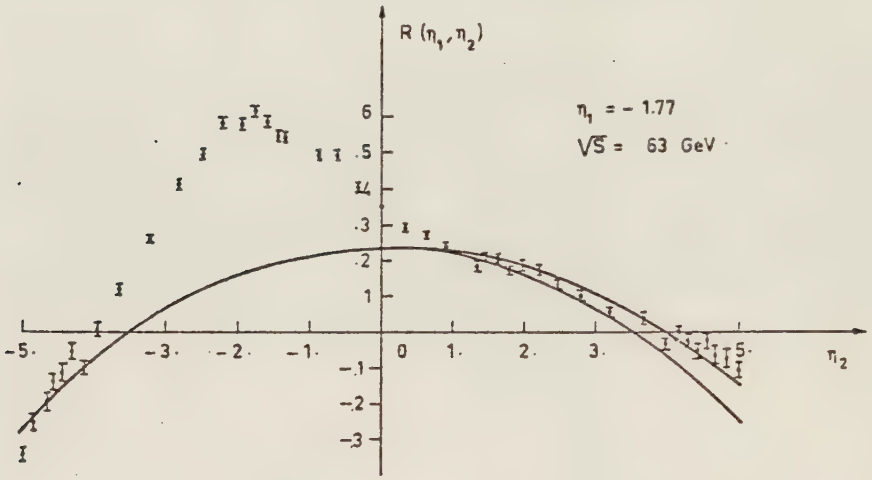

Fig. 7c.

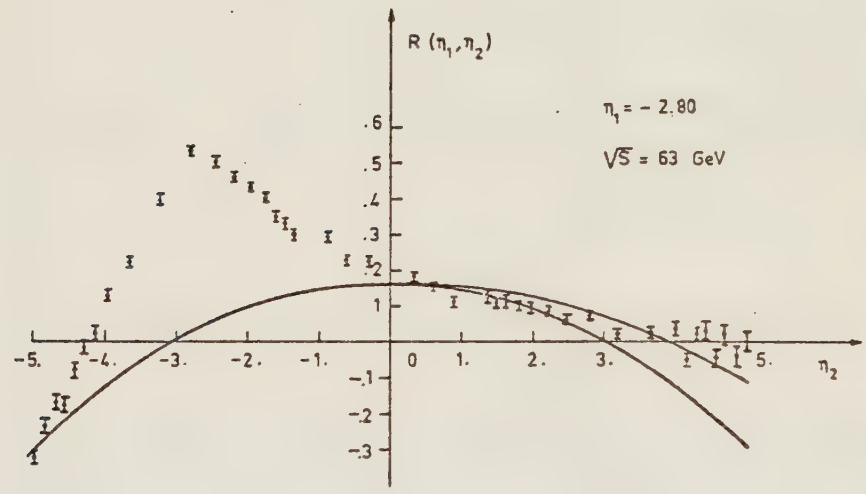

Fig. 7d.

Fig. 7. Prediction of the long range part of $R(\eta_1, \eta_2)$ at $\sqrt{s} = 63$ GeV as a function of η_2. (a) $\eta_1 = 0$, (b) $\eta_1 = -0.87$, (c) $\eta_1 = -1.77$, (d) $\eta_1 = -2.8$.

$\eta_2(g_2 \sim 0)$, and near the upper one at large $\eta_2(G_2 \sim 0)$, which improves the quality of the fit to $R(-2.8, \eta_2)$ which is entirely predicted. It is possible that for $4 < \eta \leq 5$ our upper curve is nevertheless too low; this could indicate either that our assumption $D(0) = 0$ is not perfect, or that double diffraction shows up in this region. In any case we believe that the $\alpha D/\pi$ ratio cannot be very far from our curve of fig. 6, and that we have shown that a good description of the long range part of the data can be obtained with the two component model. Insofar as this model is valid, SRO2 does not hold in terms of the η variable, since there are regions in η where the two particle cross section

is the sum of two factorizable terms which are simultaneously non zero. The next step in the investigation of diffraction and pionization is to separate D and π by a study of the one particle distribution, and to separate g and G in D. Apart from direct experimental measurement of the diffractive processes, one could also use a contour plot of $R(\eta_1, \eta_2)$ in order to extract $R(\eta, -\eta)$ from the data, and compare to eq. (10) which takes on the form

$$R(-\eta,\eta) = \alpha \frac{1-2Z-\alpha Z^2 + 2(1+\alpha)\, x(1-x)\, Z^2}{(1+\alpha Z)^2}\,, \tag{15}$$

for $2\eta \gtrsim 3$. We have set $g = xD$ and $G = (1-x)D$. Eq. (15) determines x once Z is given. For a fixed Z, R is minimum for $x = 0$ and $x = 1$, and maximum for $x = \frac{1}{2}$. Applying eq. (15) to $\eta = -4.42$ for which data also exist (but of less good quality than for smaller. $|\eta_1|$), we predict

$$-0.35 < R(-4.4, 4.4) < -0.15\,. \tag{16}$$

The experimental result is $-0.140 \pm .04$, in favour of $g \sim G$ at $\eta = -4.4$.

We thank Dr. P. Grannis for keeping us informed on the work of the Pisa-Stony Brook group and for communicating us the experimental results prior to publication.

References

[1] Pisa-Stony Brook collaboration, CERN preprint (1973).
[2] K. Gottfried, CERN preprint TH 1615 (1973);
A. Krzywicki, CERN preprint TH 1633 (1973).
[3] M. Le Bellac, Lectures at the Zakopane Summer School (June 1973).
[4] K. Fiałkowski and H.I. Miettinen, Phys. Letters 43B (1973) 64;
H. Harari and E. Rabinovici, Phys. Letters 43B (1973) 49;
J.D. Jackson and C. Quigg, NAL preprint NAL-THY-92 (1972).
[5] Pisa-Stony Brook Collaboration, submitted to the 16th Int. Conf. on high energy physics, Batavia (1972).
[6] M.G. Albrow et al., Phys. Letters 44B (1973) 207.
[7] P. Pirila and S. Pokorski, Phys. Letters 43B (1973) 502.
[8] P. Pirila and S. Pokorski, CERN preprint TH 1686 (1973);
F. Hayot and A. Morel, Saclay preprint DPh-T/73/58, to be published in Nucl. Phys. B.

REPRINT N°2

DIFFRACTION AND PRODUCTION OF UNCORRELATED PAIRS OF RESONANCES IN HIGH-MULTIPLICITY REACTIONS

LETTERE AL NUOVO CIMENTO VOL. 7, N. 6 9 Giugno 1973

Diffraction and Production of Uncorrelated Pairs of Resonances in High-Multiplicity Reactions.

F. HAYOT and A. MOREL

Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay - Gif-sur-Yvette

(ricevuto il 2 Marzo 1973)

It is known from recent experimental data on pp scattering at high energies [1-3] that the negatively charged secondaries are correlated and that the average number of neutrals depends on the number of negative particles observed. Moreover it has been noted [4] that the ratio N_c/D_c of the average number of charged particles N_c to their dispersion D_c remains remarkably constant at a value of 2 for $p_{lab} > 50$ GeV/c.

In this paper, we present a model which accounts for all of these experimental facts using only two, physically meaningful, parameters, once the average number of charged secondaries is given. We consider pp scattering, and suppose that, besides two nucleons, the final state contains essentially pions, produced partly by fragmentation of the projectile and target, and partly by some multiperipheral mechanism. In this respect our model follows the physical idea of the so-called « two component » picture which has already been proposed and studied [5]. However, we propose here a formulation of this idea which seems to us to keep closer to the underlying physics. In the models of ref. [5], one considers that the cross-section for producing n particles is the sum of two independent contributions σ_n^d and σ_n^m which account respectively for the production of n particles by diffraction and that of n particles by multiperipheralism. This means that when n secondaries are observed, *all of them* have been produced *either* by one mechanism *or* by the other. We claim that this separation does not describe the physical situation, which is rather the following: when n particles are observed, a part of them, α say, has been produced diffractively (for example those which have a rapidity larger than

---

(1) NAL (205 GeV/c): G. CHARLTON, Y. CHO, M. DERRICK, R. ENGELMANN, T. FIELDS, L. HYMAN, K. JAEGER, U. MEHTANI, B. MUSGRAVE, Y. OREN, D. RHINES, P. SCHREINER, H. YUTA, L. VOYVODIC, R. WALKER, J. WHITMORE, H. B. CRAWLEY, Z. MING MA and R. G. GLASSER: Argonne Report (1972), submitted to the *XVI International Conference on High-Energy Physics, Batavia, 1972.*

(2) ISR: G. FLUGGE, CH. GOTTFRIED, G. NEUHOFER, F. NIEBERGALL, M. REGLER, W. SCHMIDT-PARZEFALL, K. R. SCHUBERT, P. E. SCHUMACKER and K. WINTER: CERN Report (August 1972), submitted to the *XVI International Conference on High-Energy Physics, Batavia, 1972.*

(3) M. JACOB: report at the *XVI International Conference on High-Energy Physics, Batavia, 1972.*

(4) E. MALAMUD *et al.*: contribution to the *XVI International Conference on High-Energy Physics, Batavia, 1972*, quoted in ref. (3).

(5) K. FIALKOWSKI: *Phys. Lett.*, 41 B, 379 (1972); H. HARARI and E. RABINOVICI: Weizmann Institute of Science, preprint (1972); C. QUIGG and J. D. JACKSON: NAL preprint NAL-THY-93 (October 1972).

some given rapidity), and the other $y = n - \alpha$ ones have been produced along some multiperipheral chain. In terms of probabilities, this point of view states that the probability $P(n)$ for observing n particles is given by

$$P(n) = \sum_{\substack{x,\alpha \\ x+\alpha=n}} P^d(\alpha) P^m(x) , \tag{1}$$

where $P^d(\alpha)$ and $P^m(x)$ are the probabilities (which we assume to be independent) of getting respectively α fragments and x multiperipheral products. This formula is to be compared to the formulation of ref. [5]), which in terms of probabilities can be expressed as

$$P(n) = \lambda P^d(n) + (1 - \lambda) P^m(n) . \tag{2}$$

We want to stress that our separation in eq. (1) is experimentally meaningful, for example on the basis of a cut in rapidity for *each* event containing n secondaries (*).

Practically, we have to deal with two random variables n_0 and n_- (the numbers of π^0 and π^- respectively; the number of charged particles in pp scattering is $n_c = 2n_- + 2$). We set

$$\begin{cases} n_0 = x + \alpha , \\ n_- = y + \beta , \end{cases} \tag{3}$$

and now have to write the probability distributions for the various components of n_0 and n_-. Our model for these is based on the following assumptions:

i) Diffraction contributes substantially to low multiplicities only, and the diffraction mechanism is essentially energy independent. Therefore correlations, which are experimentally observed to be varying with energy, have to result from the multiperipheral mechanism.

ii) The two production mechanisms are independent, as already mentioned.

From i) and ii) it follows that the probability for getting n_0 and n_- π's is

$$P(n_0, n_-) = \sum_{\substack{x+\alpha=n_0 \\ y+\beta=n_-}} P^m(x, y) P_0^d(\alpha) P_-^d(\beta) , \tag{4}$$

where $P^m(x, y)$ is the joint probability for getting $x\,\pi^0$ and $y\,\pi^-$ along the multiperipheral chain, $P_0^d(\alpha)$ (respectively $P_-^d(\beta)$) is the probability for getting $\alpha\,\pi^0$ (respectively $\beta\,\pi^-$) by diffraction.

Since in the present work we are mainly interested in correlations due to $P^m(x, y)$, we assume for simplicity that α and β can be replaced by their average values, thus neglecting their dispersion.

iii) The nondiffractive pions arise from the decay of uncorrelated fireballs distributed along a multiperipheral chain. Hence, all the correlations occur inside each fireball.

We may imagine that the fireballs are connected to each other via pomeron exchange, so that they have vacuum quantum numbers. In this framework, an attractive way of describing the observed correlations is to consider each fireball as consisting of a pair

---

(*) We thank M. FROISSART for an illuminating discussion on these points.

of resonances (*)(**). Here we present the results for pairs of ϱ's. Other possibilities (for example, pairs of ω or A_2) will be discussed elsewhere.

If we consider an isospin-zero pair of ϱ's, the probability of it being either a $\varrho^+\varrho^-$ pair or a $\varrho^0\varrho^0$ pair is equal to $\frac{1}{2}$. Let then $p_\varrho(U, V)$ be the probability of producing U pairs (ϱ^0, ϱ^0) and V pairs (ϱ^+, ϱ^-). If the pairs are uncorrelated, we have

$$p_\varrho(U, V) = P_M(U+V)\left(\frac{1}{2}\right)^{U+V}\frac{(U+V)!}{U!\,V!}, \tag{5}$$

where $P_M(U+V)$ is the Poisson distribution for getting $U+V$ pairs, the average number of pairs being M:

$$P_M(U+V) = \frac{\exp[-M]M^{U+V}}{(U+V)!}.$$

Consequently, $p_\varrho(U, V)$ is the product of two Poisson distributions with average value $M/2$. From isospin conservation, the $U+V$ pairs decay into $x = 2V$ π^0's and $y = 2U+V$ π^-'s. Therefore the probability of getting x π^0's and y π^-'s by this mechanism is

$$P^m(x, y) = p_\varrho\left(\frac{1}{2}\left(y-\frac{x}{2}\right), \frac{x}{2}\right).$$

This formula, inserted in eq. (4), together with eq. (3), with, as explained before, α and β replaced by their average values, leads to

$$P(n_0 = x+\alpha, n_- = y+\beta) = p_\varrho\left(\frac{1}{2}\left(y-\frac{x}{2}\right), \frac{x}{2}\right). \tag{6}$$

We are now able to compute any quantity related to n_0 and n_- (and $n_c = 2n_- + 2$), as a function of M, α and β. At each energy we take $\langle n_c\rangle$ from the data [3], so that the only two parameters left are α and β, the average values of the number of π^0's and π^-'s diffractively produced. We find

$$M = \frac{\langle n_c\rangle - 2\beta - 2}{3}, \tag{7}$$

$$\langle n_-\rangle = \frac{\langle n_c\rangle - 2}{2} = \frac{3}{2}M + \beta, \tag{8}$$

$$f_2^- = \langle n_-(n_- - 1)\rangle - \langle n_-\rangle^2 = \frac{2}{3}\langle n_-\rangle - \frac{5}{3}\beta \tag{9}$$

(*) Note that this model does not demand a nonvanishing pomeron-pomeron-particle coupling. We thank G. COHEN-TANNOUDJI for this remark.

(**) Other models of resonance production of a different type have already been proposed in ref. (*) to explain the correlation between charged and neutral particles.

(*) E. L. BERGER, D. HORN and G. H. THOMAS: Argonne preprint ANL/HEP 7240 (September 1972); D. HORN and A. SCHWIMMER: Caltech preprint CALT 68-372.

and

$$\frac{N_c}{D_c} = \frac{\langle n_c \rangle}{(\langle n_c^2 \rangle - \langle n_c \rangle^2)^{\frac{1}{2}}} = \frac{\langle n_c \rangle}{\sqrt{\frac{1}{3}(\langle n_c \rangle - 2(\beta+1))}}. \tag{10}$$

Finally, we obtain for the average value of n_0 when n_- π^-'s are observed

$$\langle n_0 \rangle_{n_-} = \alpha + \frac{2(2p+1)L_p^{(-\frac{1}{2})}(-M/8)}{L_p^{(\frac{1}{2})}(-M/8)} \qquad \text{for } n_- = 2p+1+\beta, \tag{11}$$

$$\langle n_0 \rangle_{n_-} = \alpha + \frac{M}{2}\frac{L_{p-1}^{(1)}(-M/8)}{L_p^{(-\frac{1}{2})}(-M/8)} \qquad \text{for } n_- = 2p+\beta. \tag{12}$$

Here $L_p^{(\nu)}(x)$ is a Laguerre polynomial (7) of degree p and index ν.

We now proceed to discuss our results and to compare them with experiment. First, we note that if $\langle n_c \rangle$ grows logarithmically with energy, N_c/D_c only grows like $\sqrt{\log S}$, and furthermore this behaviour will emerge only at high values of $\langle n_c \rangle$ (see eq. (10)). On the contrary, f_2^- is linear in $\langle n_- \rangle$, and thus grows like $\log S$ at present energies. The numerical value $\frac{3}{4}$ of the slope in eq. (9) is model dependent: for example it would be 1 in the case where the ϱ's are replaced by ω's. As far as $\langle n_0 \rangle_{n_-}$ is concerned, the limiting behaviour will be reached only when $M/8$ will become substantially larger than 1 (see eqs. (11) and (12)). From eq. (7), one observes that this means $\langle n_c \rangle > 30$ and, for such enormous energies, $\langle n_0 \rangle_{n_-}$ behaves essentially as $2n_-$ (as can be seen by taking the asymptotic behaviour of the Laguerre polynomials in eqs. (11) and (12)). At least we can make the prediction that the slope of the curve $\langle n_0 \rangle_{n_-}$ as a function of n_- will increase slowly with energy.

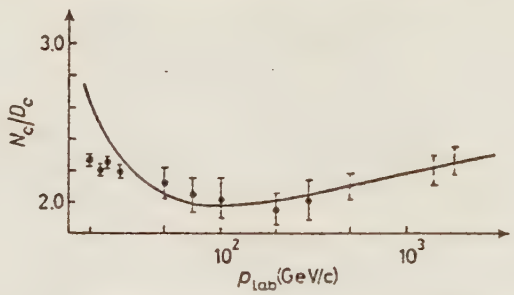

Fig. 1. – The ratio N_c/D_c for charged secondaries as a function of energy in pp reactions. Data as quoted in ref. (4). The solid line is our prediction. The three last points on this curve, which correspond to ISR energies, have (dashed) error bars due to the experimental uncertainties for $\langle n_c \rangle$.

We now turn to actually compare our results to experimental data. For f_2^- and N_c/D_c, the only parameter is β. Rather than trying a best fit to the data, we observe that β has to be such that the minimum of N_c/D_c as a function of $N_c = \langle n_c \rangle$ is about 2. From eq. (10), we find that N_c/D_c is minimum for $\langle n_c \rangle = 4(\beta+1)$, and this minimum is equal to 2 for $\beta = \frac{3}{4}$. Our results with this value for β are shown in Fig. 1 and 2 for N_c/D_c and f_2^- respectively. We obtain a good agreement with the data. The only deviation occurs for the lowest multiplicities, for which we have to remember that M,

(7) G. SZEGO: *Orthogonal Polynomials*, American Mathematical Society Colloquium Publication, Vol. 23 (Providence, R. I., 1939).

the average number of ρ pairs, is still very small. For the same reason, we do not expect to account for the f_3^- data ([3]). At present available energies ($p_{lab} < 300$ GeV/c), $\langle n_- \rangle$ is less than of the order of 3 so that it is not expected that f_3^- has reached its

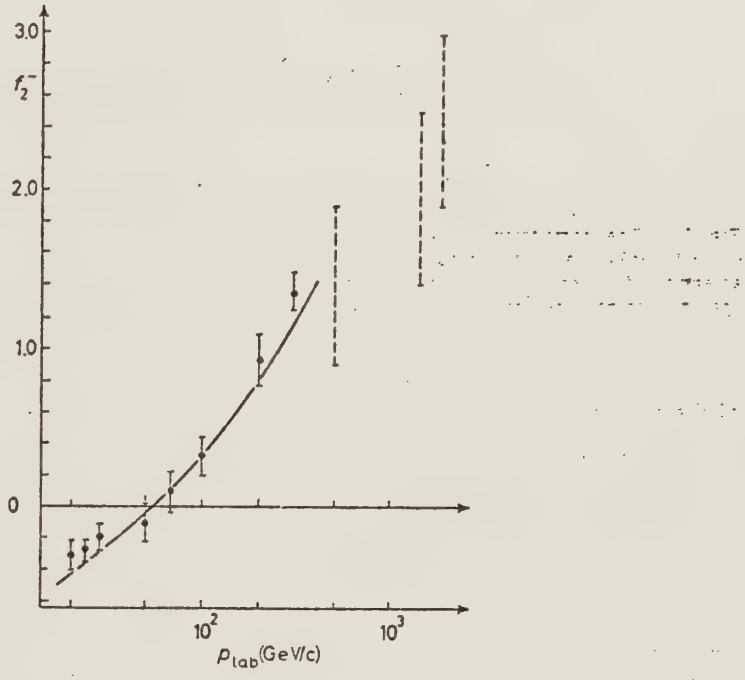

Fig. 2. – The correlation coefficient f_2^- for negative secondaries as a function of energy in pp reactions. Data as quoted in ref. ([3]). The solid line is our prediction. The three last points on this curve, which correspond to ISR energies, have (dashed) error bars due to experimental uncertainties for $\langle n_c \rangle$.

asymptotic regime. We have however to keep in mind that, in the ρ pair model, $3\pi^-$ correlations are only of diffractive type and thus constant with energy. The same would be true for an ω pair model, but not for an A_2 pair model, where the $A_2^+ A_2^-$ decay mode of a fireball yields $3\pi^-$ correlations. As to our results for $\langle n_0 \rangle_{n_-}$ (eqs. (11) and (12)), they are shown on Fig. 3 *a*) and *b*). Notice the amusing fact that, in our model, $\langle n_0 \rangle_{n_-}$ depends on the parity of the number of multiperipheral pions. This particular feature, which disappears with increasing energy, could hopefully explain irregularities which are present in the 205 GeV/c data and not at ISR energies (see Fig. 3 *a*) and *b*)). However differences due to the parity of multiperipheral π^-'s can be washed out, if the dispersion of the diffractive π^-'s, which has been ignored here, is not sufficiently smaller than 1. In fact, in order to be able to compare our predictions to experimental data at each integer value of n_-, we cannot simply replace β by its average $\frac{2}{3}$. For definiteness, our curves in Fig. 3 are drawn with $P_-^d(0) = \frac{1}{3}$, $P_-^d(1) = \frac{2}{3}$ in eq. (4). This corresponds to a dispersion d^2 for β equal to $\frac{2}{9}$. One observes in Fig. 3 *a*) and *b*) that oscillations are visible at 205 GeV/c, but not at ISR energy. Our second and last parameter α is not well determined by the data. Curves are drawn for $\alpha = 1$.

As a conclusion, we would like to insist upon a few important points:

i) The high-energy behaviours $f_2^- \propto \langle n_- \rangle$ and $N_c/D_c \propto \sqrt{\langle n_- \rangle}$ are independent of the particular model for the decay of uncorrelated multiperipheral fireballs.

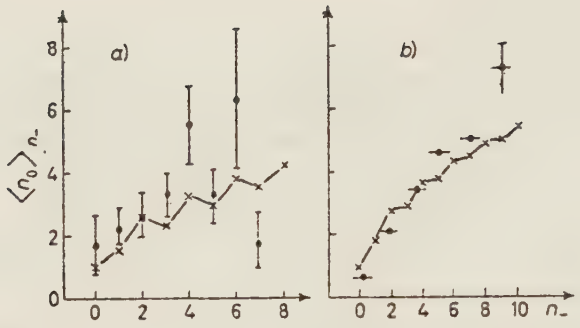

Fig. 3. – The correlation $\langle n_0 \rangle_{n_-}$ between π^0's and π^-'s at *a*) 205 GeV/c, and *b*) 1400 GeV/c. $\sqrt{S} = 53$ GeV. Data as quoted in ref. (4). Theoretical points are predictions with $\alpha = 1$, and $P^d(\beta)$ as explained in the text. The broken lines between theoretical points are drawn to guide the eye. Note that oscillations are present at 205 GeV/c between even and odd values of n_-.

ii) The linearity of f_2^- as a function of $\langle n_- \rangle$ is a consequence of our formulation of the two-component picture, in contrast with that of ref. (5) which yields $\langle n_- \rangle^2$ terms.

iii) A precise measurement of the coefficient of $\langle n_- \rangle$ in f_2^- would give more information about the nature of the pairs of resonances. So would f_3^- measurements at high enough multiplicities. The dispersion of the number of π^0's for a fixed number of π^-'s depends also on the model considered: it is zero for $\omega\omega$ production, where each π^- is accompanied exactly by one π^0 (*).

iv) More information about the separation of the production mechanism into two components can be experimentally obtained by looking at the rapidity pattern in individual events. Take all events with n_- negative tracks, count the number β of negative particles with rapidity larger than some given rapidity Y_0, and make the histogram $\sigma_{n_-}(\beta, n_- - \beta)/\sigma_{in}$, which has to be compared to a product of probability distributions $P^d(\beta)P^m(n^- - \beta)$, as explained above. A candidate for the value of Y_0 could be the half-width of the rapidity plateau in the inclusive reaction $pp \to \pi^- + X$.

v) Our comparison with the data could indicate that the slopes of $\langle n_0 \rangle_{n_-}$ *vs.* n_- and of f_2^- *vs.* $\langle n_- \rangle$ are a little bit too small. This is an argument in favour of a certain amount of ω pair production. In this context, an attractive generalization of our model is a multiperipheral production of pairs of vector mesons, each pair belonging to an SU_3 singlet, since produced by pomeron exchange.

(*) This remark has already been made by D. HORN and A. SCHWIMMER, ref. (6), in the case of single-ω production.

LETTERE AL NUOVO CIMENTO VOL. 8, N. 1 1 Settembre 1973

Diffraction and Production of Uncorrelated Pairs of Resonances in High-Multiplicity Reactions.

F. HAYOT and A. MOREL

Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay - Gif-sur-Yvette

(*Lett. Nuovo Cimento*, 7, 221 (1973))

If one considers an isospin-zero pair of ρ's, the probability of having a $\rho^+\rho^-$ pair is $\frac{2}{3}$ and that of a $\rho^0\rho^0$ pair is $\frac{1}{3}$, and not $\frac{1}{2}$ in both cases as written in our article. Our error leads to trivial changes in our formulae from formula (7) onwards, in particular the denominator in (7) is now $\frac{8}{3}$ and not 3, the coefficient of $\langle n_- \rangle$ in (9) is $\frac{1}{2}$ and not $\frac{2}{3}$, and the parameter $\beta = \frac{1}{2}$ and not $\frac{2}{3}$. As a consequence, the agreement of f_2^- with the data is not quite as good, but that of $\langle n_0 \rangle_{n^-}$ is better. More importantly, however, none of the qualitative features of our model is changed and except for point v), the important points made in our final discussion still hold.

* * *

We thank Dr. LE BELLAC for making us aware of our initial mistake.

REPRINT N°3

FIT TO THE N-PARTICLE CROSS SECTIONS IN A MODEL WITH SU(3) SINGLET UNCORRELATED FIREBALLS

REPRINT N°3

Nuclear Physics B59 (1973) 493–510. North-Holland Publishing Company

FIT TO THE *N*-PARTICLE CROSS SECTIONS IN A MODEL WITH SU(3) SINGLET UNCORRELATED FIREBALLS

F. HAYOT and A. MOREL
Service de Physique Theorique, Centre d'Etudes Nucleaires de Saclay, 91190, Gif-sur-Yvette, France

Received 21 February 1973
(Revised 10 May 1973)

Abstract: We reproduce the cross sections σ_{n_c} for n_c charged particle production in high energy pp scattering, and thus obtain quasi-scaling for the KNO function $\langle n_c\rangle\sigma_{n_c}/\sigma_{\text{inel}}$ over the energy range 50–300 GeV/c. This result is obtained in a model in which the multiperipheral secondaries originate from a chain of uncorrelated fireballs, decaying into pairs of octet vector mesons in an SU(3) singlet state, and the secondaries of diffractive type follow a Poisson distribution. At each energy, the average number $\langle n_c\rangle$ of charged secondaries is taken from experiment. Then all cross sections are obtained using only one free parameter.

1. Introduction

In this work we study n-particle total cross sections in high-energy pp collisions. Two features of the data [1] are of special interest: first the fact that the quantity $\langle n\rangle\sigma_n/\sigma_{\text{inel}}$ appears to depend essentially upon $n/\langle n\rangle$ (KNO scaling [2]), secondly the existence of correlations among charged secondaries that increase with energy. The basic idea of our study has already been presented in a recent paper [3]. We consider that each n-particle event is made up of particles of two different origins: one fraction of these n particles results from the fragmentation of either target or projectile, the remaining ones result from some multiperipheral chain and mainly populate the central region. In the model we will consider the multiperipheral chain is linked to the incoming protons *via* pomeron exchange. For that reason we call diffractive the particles that result from fragmentation. From our point of view, then, one type of diagrams only, with particles both of multiperipheral and diffractive origin, is relevant. As a consequence the probability of producing n particles appears in our case as a convolution of two probabilities (see eq. (3.2)), one we call the diffractive component, the other one being the multiperipheral component. This is very different from the usual so-called two-component picture [4], in which that same probability is written as the *sum* of two terms, one where *all* particles are diffractively produced,

the other one where none is and the n particles originate from a usual multiperipheral diagram. In our picture, we give a very explicit description of the multiperipheral component which we assume is the only responsible for the energy dependence of such quantities as average charged multiplicities and correlations. This description involves an energy-dependent number of uncorrelated fireballs, each one decaying into a well defined number of particles.

In sect. 2 we give a qualitative discussion of correlations which leads us to the above mentioned picture for the multiperipheral chain. In sect. 3, this picture is made more precise and general expressions for the Mueller correlation coefficients [5] f_q are given, which show that all of them are linear functions of $\langle n \rangle$. The general scheme obtained is then applied in sect. 4 to a specific model for the production of fireballs. We imagine that multi-pomeron exchange is the basic process for their production. Therefore they are all SU(3) singlets which we take to decay into pairs of octet vector mesons. This is a generalization of the simple model already presented in ref. [3]. With the help of only one parameter, namely the average number of particles in the diffractive component, we are able to reproduce all σ_n cross sections, the correlations between charged and neutral particles, and to give a semi-quantitative estimate of the relative multiplicities of π's and K's. Our result for σ_n is presented by means of the KNO plot (fig. 1). It shows that between 50 and 300 GeV/c, the quantity $\langle n \rangle \sigma_n / \sigma_{\text{incl}}$ is practically energy independent: this fact occurs in our model as a kind of compensation between the two production mechanisms (sect. 4). Sect. 5 is devoted to a general discussion of our results and some problems related to our model.

2. Qualitative discussion of correlations

It is interesting to look first in some detail at the correlations between charged particles that are simply due to charge conservation. Suppose we know the probability distribution $P_-(n_-)$ for the negative tracks. Let Q be the total charge of the initial state. The number n_c of charged tracks for n_- negative ones is

$$n_c = 2n_- + Q . \tag{2.1}$$

We have of course

$$\langle n_c \rangle = 2\langle n_- \rangle + Q , \tag{2.2}$$

$$\langle n_c^2 \rangle = 4\langle n_-^2 \rangle + 4Q\langle n_- \rangle + Q^2 . \tag{2.3}$$

Let us compute the dispersion D_c^2 of n_c as a function of f_2^-, where

$$f_2^- = \langle n_-(n_- - 1) \rangle - \langle n_- \rangle^2 . \tag{2.4}$$

We obtain

$$D_c^2 = 4f_2^- + 2\langle n_c\rangle - 2Q\,, \tag{2.5}$$

or equivalently

$$f_2^c = D_c^2 - \langle n_c\rangle = \langle n_c\rangle + 4f_2^- - 2Q\,. \tag{2.6}$$

At this point we want to make two trivial remarks:

(i) If there is no correlation at all between the negative particles ($f_2^- = 0$), then f_2^c is *linear as a function of* $\langle n_c\rangle$ with slope unity. It is therefore increasing with energy the same way as $\langle n_c\rangle$ does, and it may be negative at low energies if $\langle n_c\rangle$ is sufficiently small.

(ii) It is fashionable to consider the quantity

$$Z_c = \frac{\langle n_c\rangle}{\sqrt{D_c^2}}\,, \tag{2.7}$$

which is experimentally observed to remain essentially constant around two for pp scattering. Consider again the case where $f_2^- = 0$. From eq. (2.5), we have

$$Z_c^2 = \tfrac{1}{2}\langle n_c\rangle + \tfrac{1}{2}Q + \frac{Q^2}{2(\langle n_c\rangle - Q)}\,, \tag{2.8}$$

The point we want to make is that, as a function of $\langle n_c\rangle$, Z_c^2 is stationary for $\langle n_c\rangle = 2Q$ (= 4 for pp scattering). So, Z_c^2 is expected to vary slowly with energy, not only because $\langle n_c\rangle$ varies slowly with energy, but also because $\mathrm{d}Z_c/\mathrm{d}\langle n_c\rangle \simeq 0$ for $\langle n_c\rangle \simeq 4$. This results from a cancellation between the increase of $\frac{1}{2}\langle n_c\rangle$, where the factor $\frac{1}{2}$ can be traced back to the factor 2 in eq. (2.1) (correlation by *pairs*), and the decrease of $Q^2/2(\langle n_c\rangle - Q)$, (conservation of the initial charge).

According to remark (i), f_2^c has, in the absence of any correlation between negative particles, the very behaviour which is experimentally observed for f_2^-. This is due to the factor 2 in eq. (2.1). Therefore we are lead to the following proposal for n_-: the number n_- of negative secondary particles is the sum of two terms

$$n_- = \lambda y + \beta\,, \tag{2.9}$$

where we consider that y objects are produced *independently*, each of them decaying into λ negative particles, and that β other negative particles are produced from another type of mechanism. Eq. (2.9) for n_- is the exact reproduction of eq. (2.1) for n_c, with the only difference that Q is a fixed number for each reaction, whereas β has to be considered here as a random variable. We now write for the probability distribution $P_-(n_-)$

$$P_-(n_-) = \sum_{y,\beta} \delta_{n_-,\lambda y+\beta} P(y,\beta)\,, \tag{2.10}$$

where $P(y, \beta)$ is the probability distribution for the two random variables y and β, and $\delta_{i,j}$ the Kronecker delta symbol. If we assume these two variables to be independent (which we shall do in the following), we obtain

$$P_{-}(n_{-}) = \sum_{y,\beta} \delta_{n_{-},\lambda y+\beta} P_1(y) P_2(\beta) , \tag{2.11}$$

with P_1 and P_2 the respective distributions for y and β. Since the y objects are produced independently, we shall use for P_1 a Poisson distribution with average M, namely

$$P_1(y) = \frac{e^{-M} M^y}{y!} . \tag{2.12}$$

From eqs. (2.9), (2.11) and (2.12), we obviously have

$$\langle n_{-} \rangle = \lambda M + \langle \beta \rangle . \tag{2.13}$$

Let f_2^{β} be the usual correlation coefficient for β

$$f_2^{\beta} = \langle \beta^2 \rangle - \langle \beta \rangle^2 - \langle \beta \rangle .$$

In terms of f_2^{β}, $\langle \beta \rangle$ and $\langle n_{-} \rangle$ the correlation coefficient f_2^{-} is found to be

$$f_2^{-} = (\lambda - 1) \langle n_{-} \rangle - (\lambda - 1) \langle \beta \rangle + f_2^{\beta} . \tag{2.14}$$

f_2^{-} is linear as a function of $\langle n_{-} \rangle$ exactly as f_2^{c} is as a function of $\langle n_c \rangle$ in eq. (2.6) when f_2^{-} is zero.

From this discussion, we conclude that a fit to the data for f_2^{-} with a linear form in $\langle n_{-} \rangle$ yields two physically meaningful parameters. One is λ, which can be interpreted as *the average value of the number of negative particles contained in each independently emitted object.* The other parameter $\lambda(\beta - 1) - f_2^{\beta}$ yields information on the probability distribution of the second variable β, once λ is known.

3. General formulation of uncorrelated fireball models

In the spirit of the preceding section, and in order to reproduce the general form of f_2^{-} (eq. (2.14)), we now consider a model in which the y objects of sect. 2 are uncorrelated fireballs, linked together *via* pomeron exchange along some multiperipheral chain, and which decay subsequently into secondary particles through *pairs* of resonances. In ref. [3] we presented the case for ρ resonances yielding pions. The β particles in eq. (2.9) are then supposed to result from a diffractive mechanism. Despite our crude treatment of this mechanism in ref. [3], we found that our ρ model gave remarkable agreement with experimental results for correlations between nega-

tively charged secondaries, and between charged and negative ones. Let us repeat here that our model is based on a two-component picture for the produced particles. But as stressed before (ref. [3]), this picture (as presented in eqs. (2.10) or (2.11)) is very different from the usual one (ref. [4]), for which each n-particle event is exclusively either of multiperipheral or of diffractive origin. As a consequence, our final distributions always appear as *convolutions* of the two components and never as *sums* of them.

Let us first concentrate upon the multiperipheral component, denoted by the superscript m in all formulae. The general class of models we consider, namely production of uncorrelated pairs of resonances in an SU(2) (or SU(3), see below) singlet state, has some general properties, which happen to be extremely simple and remarkable. The general scheme is as follows: *uncorrelated* fireballs are produced in SU(2) (or SU(3)) singlet states; they decay into various *independent* pairs of resonances ($\rho_0\rho_0$, $\rho_+\rho_-$, $\omega\omega$ for example), which themselves subsequently decay *independently* into the final states we are interested in ($2\pi^+2\pi^-$, $2\pi^0\pi^+\pi^-$, $2\pi^+2\pi^-2\pi^0$ respectively for the above example). As a consequence of this cascade of independent production and decays, the probability distribution for getting n_1 states of type 1, n_2 states of type 2, etc. ($n_1 \times (2\pi^+2\pi^-)$, $n_2 \times (2\pi^0\pi^+\pi^-)$, $n_3 \times (2\pi^+2\pi^-2\pi^0)$ in the above example), is

$$P^{(m)}(n_1, n_2, \ldots) = P_{\langle n_1\rangle}(n_1) \times P_{\langle n_2\rangle}(n_2) \times \ldots , \tag{3.1}$$

where $P_{\langle n_1\rangle}, P_{\langle n_2\rangle}, \ldots$ are Poisson distributions with average value $\langle n_1\rangle, \langle n_2\rangle, \ldots$.

Let now n be the number of charged or negative secondaries. The correlation functions $f_q^{(m)}$ for n particles, as defined by Mueller [5], have the following properties (see appendix A) within the class of the models we consider:

(i) $f_q^{(m)}$ is linear in $\langle n\rangle$,

(ii) the slope of $f_q^{(m)}$ *versus* $\langle n\rangle$ is positive,

(iii) $f_q^{(m)}$ is zero whenever $q > n$.

All of the preceding considerations were concerned with the multiperipheral part of the production mechanism. According to eq. (2.11), we have to convolute the distribution for the multiperipheral events with that of the diffractively produced particles. Let β be the random variable associated with the number of diffractive negative tracks, and $P^d(\beta)$ the corresponding distribution. We make the following assumption: the diffractive mechanism is *energy independent*, and $P^d(\beta)$ is a *Poisson distribution* with $B = \langle\beta\rangle$. Our main reason for making this choice is simplicity. In particular, our choice is compatible with the reasonable idea that both fragmentation of the target and projectile contribute independently to the diffractive process. In any case, most of the results which are presented in this paper, do not depend too much on this simple choice.

We are now able to give the general formulae for the probability distributions and the correlation function for the number n_c of charged particles, which we will apply to a specific model in the next section.

For pp scattering, the probability distribution is

$$P_r(n_c) = \sum_{n_i,\,\beta} P_B(\beta) \prod_i P_{\langle n_i\rangle}(n_i)\,, \tag{3.2}$$

where

$$P_m(x) = \frac{e^{-m} m^x}{x!}, \quad \text{(Poisson distribution)},$$

and the summation runs over all n_i and β such that

$$n_c = 2\beta + 2 + 2\sum_i \lambda_i n_i\,. \tag{3.3}$$

Here λ_i is the number of negative particles contained in the final state i of the decay of one fireball. If p_i denotes the probability that one fireball decays into the state i, we have

$$\langle n_i\rangle = p_i M\,, \tag{3.4}$$

where M is the average number of multiperipheral fireballs. We have of course

$$M = \frac{\langle n_c\rangle - 2B - 2}{2\sum_i \lambda_i p_i}. \tag{3.5}$$

Once $\langle n_c\rangle$ is taken from experiment, λ_i and p_i are given by the particular model chosen for the fireballs. In the next section, we will see that B is our only free parameter.

From eqs. (3.2) and (3.3), it follows that n_c is the sum of the independent random variables 2β, 2, and $2\lambda_i n_i$. Taking into account that the distributions for β and n_i are of Poisson type, and using eqs. (A.7), (A.9) and (A.10) of appendix A, we find for the Mueller correlation functions for n_c the general expression

$$f_q^c = 2B\delta_{q,2} + 2(-1)^{q-1}(q-1)! + \sum_i 2\lambda_i(2\lambda_i - 1)\ldots(2\lambda_i - q + 1)\langle n_i\rangle\,, \tag{3.6}$$

where $\langle n_i\rangle$ is related to $\langle n_c\rangle$ through eqs. (3.4) and (3.5). $\delta_{q,2}$ is a Kronecker delta function. Expression (3.6) for f_q^c shows the general properties mentioned above, in particular the linear behaviour as a function of $\langle n_c\rangle$. It shows moreover another remarkable fact, namely that the correlation coefficients for each i are additive. This is due to the mutual independence of the processes leading to a given final state i. It is clear too, that for the same reason, the correlation coefficients due to the multiperipheral and diffractive mechanisms add.

These properties will now be discussed in the particular model where the fireballs are assumed to be SU(3) singlets decaying into pairs of octet vector mesons.

4. SU(3) singlet fireballs decaying into pairs of octet vector mesons

We now consider the model in which the fireballs, which are linked together through pomeron exchanges, decay into pairs of octet vector mesons in an SU(3) singlet state. These pairs are $\rho\rho$, $\omega\omega$, $\varphi\varphi$ and $K^*\bar{K}^*$. They decay into 13 configurations of charged and neutral π's and K's. The relative branching ratios for all these final decay products, i.e. the probabilities p_i of these 13 states i in the decay of one fireball, are easily calculated from the SU(3) and SU(2) Clebsch-Gordan coefficients and the $\omega-\varphi$ mixing angle, which is chosen here to be 45°. The list of the states i, and their probabilities, are given in table 1. From this table, one can find the constants λ_i and p_i which are necessary for using eqs. (3.3), (3.4) and (3.6), or the equivalent equations, if one is interested in another type of particles than the charged ones. The only unknown quantity we are left with, once $\langle n_c \rangle$ is taken from experiment, is the energy independent parameter B, the average number of diffrac-

Table 1
List of final states into which decay the pairs of vector meson resonances forming SU(3) singlets and the probabilities for each state as computed from SU(2) and SU(3) Clebsch-Gordan coefficients

Resonance pairs	Final states	Probabilities
$\rho\rho$	$2\pi^+2\pi^-$	$\frac{1}{8}$
	$2\pi^0\pi^+\pi^-$	$\frac{1}{4}$
$\omega\omega$	$2\pi^+2\pi^-2\pi^0$	$\frac{1}{16}$
$\varphi\varphi$	$2K^02\bar{K}^0$	$\frac{1}{32}$
	$2K^+2K^-$	$\frac{1}{32}$
$K^*\bar{K}^*$	$2\pi^02K^0$	$\frac{1}{36}$
	$\pi^0\pi^-\bar{K}^0K^+$	$\frac{2}{36}$
	$\pi^0\pi^+K^0K^-$	$\frac{2}{36}$
	$\pi^+\pi^-K^+K^-$	$\frac{4}{36}$
	$\pi^0\pi^+K^0K^-$	$\frac{2}{36}$
	$\pi^+\pi^-K^0\bar{K}^0$	$\frac{4}{36}$
	$2\pi^0K^+K^-$	$\frac{1}{36}$
	$\pi^0\pi^-\bar{K}^0K^+$	$\frac{2}{36}$

tively produced negative particles. In order to determine B, we use the experimental fact that the quantity Z_c^2 (see eq. (2.7)) is practically constant and equal to four as a function of energy for $50 < p_{lab} < 300$ GeV/c. Using eqs. (3.4), (3.5) and (3.6), we compute Z_c^2 as a function of $\langle n_c \rangle$ and B. Writing that Z_c^2 has a minimum in $\langle n_c \rangle$ and that at this minimum it has the value four, we determine B to be

$$B = \frac{\sum_i \lambda_i^2 p_i}{\sum_i \lambda_i p_i} . \tag{4.1}$$

We recall that λ_i is the number of *negative* particles present in state i. This number can be read off table 1. Eq. (4.1) is a very important one: it relates the distribution of the negative particles in the diffraction mechanism (*via* the parameter B), to their distribution inside each multiperipheral fireball. If q_- is the number of negative particles inside a fireball, the equation (4.1) reads

$$B = \frac{\langle q_-^2 \rangle_f}{\langle q_- \rangle_f} . \tag{4.2}$$

We can now proceed to the evalutation of the distributions and correlations we are interested in.

4.1. Distribution of charged particles and the KNO plot

By inspection of table 1, one observes that from the point of view of charged particles, the thirteen states i can be grouped into two classes (but for channels 4 and 6): those which contain two charged particles (class 1) and those which contain four charged particles (class 2). Using the fact that sums of Poisson variables are Poisson variables, we can replace eqs. (3.2) and (3.3) by

$$P_r(n_c) = \sum_{Y_1 + 2Y_2 = \frac{1}{2}(n_c - 2)} P_{\langle Y_1 \rangle}(Y_1) P_{\langle Y_2 \rangle}(Y_2) , \tag{4.3}$$

with

$$\begin{aligned} \langle Y_1 \rangle &= B + \sum_{(1)} \langle n_i \rangle , \\ \langle Y_2 \rangle &= \sum_{(2)} \langle n_i \rangle , \end{aligned} \tag{4.4}$$

the summation $\Sigma_{(1)}$ (resp. $\Sigma_{(2)}$) running over those indices i which correspond to states of class 1 (resp. class 2). After some manipulations [6], we find easily, using eqs. (3.4), (3.5) and (4.1), (4.3) and (4.4) and table 1,

$$P_r(n_c=4p+2) = \exp(-\langle Y_1\rangle - \langle Y_2\rangle)(4\langle Y_2\rangle)^p \frac{p!}{(2p)!} L_p^{-\frac{1}{2}}\left(-\frac{\langle Y_1\rangle^2}{4\langle Y_2\rangle}\right), \tag{4.5a}$$

$$P_r(n_c=4p+4) = \exp(-\langle Y_1\rangle - \langle Y_2\rangle)(4\langle Y_2\rangle)^p \frac{p!}{(2p+1)!} L_p^{\frac{1}{2}}\left(-\frac{\langle Y_1\rangle^2}{4\langle Y_2\rangle}\right)\langle Y_1\rangle, \tag{4.5b}$$

where $L_p^{\gamma}(x)$ is the Laguerre polynomial of degree p and index γ, and where explicitly

$$\langle Y_1\rangle = \frac{q_1\langle n_c\rangle + 4q_2B - 2q_1}{2(q_1+2q_2)},$$

$$\langle Y_2\rangle = \frac{q_2(\langle n_c\rangle - 2B - 2)}{2(q_1+2q_2)}, \tag{4.6}$$

$$B = \frac{q_1+4q_2}{q_1+2q_2}, \tag{4.7}$$

with

$$q_1 = \tfrac{176}{288}, \qquad q_2 = \tfrac{95}{288}. \tag{4.8}$$

Thus eqs. (4.5) to (4.8) give us the expression for $\sigma_{n_c}/\sigma_{\text{inel}}$ as a function of $\langle n_c\rangle$ which we take from experiment. We compare our result to experimental data by plotting the quantity $\psi_c = \langle n_c\rangle \sigma_{n_c}/\sigma_{\text{inel}}$ *versus* the variable $n_c/\langle n_c\rangle$. The ψ_c is the so-called KNO function [2]. The comparison is presented in fig. 1. The agreement of our fit with the experimental points is found to be excellent. We want to stress the remarkable fact that, *though our model does not yield* ψ_c *as a function of* $n_c/\langle n_c\rangle$, we are unable *to draw separate curves for energies ranging from* 50 *to* 300 GeV/c. In fig. 1, we show the expected curve for the maximum ISR energy ($\sqrt{s}$ = 53 GeV, dashed line), which is seen to be slightly different from the preceding ones, but only for $n_c/\langle n_c\rangle > 2$. Our model is therefore different from the simple one proposed by Lévy [8], in which the number of particles per fireball is energy dependent, whereas the average number of fireballs is not. That model leads to strict KNO scaling, whereas ours gives quasi-scaling in the 50–300 GeV/c range. Both models fit equally well the experimental data. The quasi-scaling property of our model can be understood in another way by looking at the moments of the charged particle distribution.

The simplest way of computing the moments $\langle n_c^q\rangle$ in our model is to start from the f_q correlation coefficients as given by eq. (3.6) and formula (A.11) of the appendix. Our results for $\langle n_c^q\rangle/\langle n_c\rangle^q = c_q$ are presented in table 2 for energies between 50

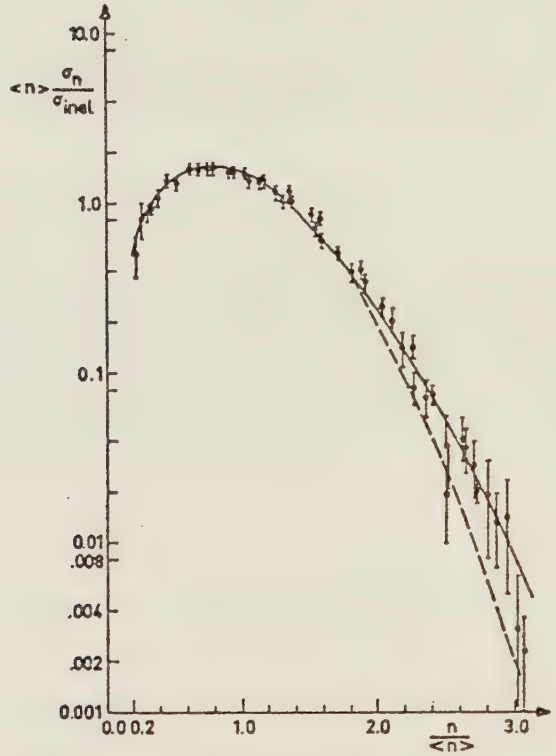

Fig. 1. SU(3) model fit to the quantity $\langle n\rangle\sigma_n/\sigma_{\text{inel}}$ as a function of $n/\langle n\rangle$ for pp → charged particles, for energies 50 GeV/c, 69 GeV/c, 102 GeV/c, 205 GeV/c, 303 GeV/c. Data from ref. [1]. The dashed line (- - - - - -) is our prediction at 1500 GeV/c ($\sqrt{s}$ = 53 GeV). It is distinguishable from the full line (————) only when $n/\langle n\rangle > 2$. However, we have drawn it for $\langle n_c\rangle = 12.5$ without taking into account the error bar on $\langle n_c\rangle$.

and 300 GeV/c and for $q = 2, 3, 4$. They are compared to experimental data, as quoted by Slattery [7]. The agreement we find is not surprising since we already know that our full distribution fits the data. The point we want to make is that *the c_q's are remarkably constant* inside the considered energy range, which reflects the fact that our distribution gives quasi KNO scaling. Why is this so? For simplicity we make the argument on c_2, which can be written as

$$c_2 = 1 + \frac{1+\alpha}{\langle n_c\rangle} - \frac{2[(\alpha-1)B+\alpha+1]}{\langle n_c\rangle^2}, \tag{4.9}$$

where α stands for the coefficient of $\langle n_c\rangle$ in f_2 (see eqs. (3.4), (3.5) and (3.6)). Within the framework of our model, α (which is related to the distribution of particles in each fireball) and B are constant with energy, Furthermore, α is larger than 1 (there is on the average more than 1 negative particle per fireball). Thus, according to our model

Table 2
Coefficients $c_q = \langle n^q \rangle / \langle n \rangle^q$ for charged particles, with $q = 2, 3, 4$ for energies between 50 GeV/c and 1500 GeV/c, and in parentheses the available experimental results with error bars as reported in ref. [7]

q	P_{lab} (GeV/c)							
	50	69	102	205	303	500	1000	1500
2	1.236	1.247	1.250	1.245	1.234	1.226	1.210	1.194
	1.236 ± 0.014	1.2415 ± 0.0084	1.249 ± 0.014	1.258 ± 0.019	1.245 ± 0.015			
3	1.797	1.838	1.849	1.827	1.780	1.753	1.690	1.632
	1.784 ± 0.054	1.806 ± 0.030	1.828 ± 0.052	1.856 ± 0.065	1.816 ± 0.051			
4	2.974	3.100	3.130	3.055	2.912	2.831	2.651	2.492
	2.89 ± 0.15	2.959 ± 0.084	3.01 ± 0.15	3.08 ± 0.18	2.99 ± 0.14			

(i) ultimately, c_2 will tend to 1, *but*

(ii) there is a compensation between the two additional terms which is such that c_2 is flat over a very wide range of energies.

It is very easy to convince oneself that these features are general for all c_q's. Our conclusion concerning this point is that *the observed scaling or quasi-scaling arises from a competition between the two production mechanisms.* Note that if in eq. (4.9) we let B and α grow with energy proportionally to $\langle n_c \rangle$ we can realize exact asymptotic scaling. This kind of model would be of the kind considered in ref. [8], where the number of particles per fireball increases with energy.

4.2. Kaons and neutral pions in high-energy reactions

Table 1 allows one to compute the multiperipheral part of the distribution for any kind of particle. Let us first compute the average relative amount of charged K's among all charged particles in the *multiperipheral* chain. Simple counting leads to

$$\tau_m^{K_c} = \frac{\langle n_{K_c} \rangle_m}{\langle n_c \rangle_m} = 25\% . \tag{4.10a}$$

The experimental number for $\tau^{K_c} = n_{K_c}/n_c$ at ISR energies is about 10% (ref. [10]). This should not be taken as a too serious discrepancy with our prediction, which is

valid for the *multiperipheral part* of the production only. Suppose for example that only the excess of K^+ *versus* K^- ($\langle n_{K^+}\rangle - \langle n_{K^-}\rangle \simeq 0.1$, independently of the energy [10]), is of diffractive origin as far as the production of charged K's is concerned, then we have

$$\tau^{K_c} = \frac{\tau_m^{K_c} + 0.1/\langle n_c\rangle_m}{1 + (2B+2)/\langle n_c\rangle_m}. \tag{4.10b}$$

In our model $2B+2 = 3 + 2$ and correspondingly at ISR energies $\langle n_c\rangle_m \simeq 6$. Thus we obtain

$$\tau^{K_c} \simeq \frac{0.18 + 0.02}{1.9} = 15\%,$$

which is not so high. In any case, eq. (4.10b) is at least a qualitative interpretation of the observed result [10] that the relative average multiplicity of K_c increases with energy because $\langle n_c\rangle_m$ does. Note also that, in our model, about $\frac{1}{5}$ of the charged K's are produced through a $\varphi\varphi$ channel (table 1), a system which is substantially heavier than the $\rho\rho$, $K^*\bar{K}^*$ and $\omega\omega$ ones, so that this channel could be partly suppressed if the fireballs have not high enough masses.

In the same way as for the charged K's, we also find for K_S^o, which can be seen in bubble chamber experiments

$$\frac{\langle K_S^o\rangle_m}{\langle n_c\rangle_m} = 12\% \tag{4.11a}$$

and for π_o's, compared to negative secondaries

$$\frac{\langle \pi_o\rangle_m}{\langle n_-\rangle_m} = 75\%. \tag{4.11b}$$

As we did in the $\rho\rho$ model of ref. [3], we also present our prediction for the average number $\langle \pi_o\rangle_{n_-}$ of π_o's for a given number n_- of negative tracks. We again assume that there are no correlations between neutrals and charged particles in the diffraction products, which would not be correct if resonances play an important role in this mechanism. Curves are presented in fig. 2 for an average number of (uncorrelated) π_o's in the fragmentation region equal to 1. The calculation is straightforward, though tedious. Table 1 gives the number of π_o's in each of the states i, and one writes an equation of type (3.2) for the probability $\Pr(n_o, n_-)$ for finding exactly $n_o\pi_o$'s and $n_-\pi_-$'s. We find

$$\langle \pi^o\rangle_{n_-} = \frac{\sum_{n_o} n_o \Pr(n_o, n_-)}{\Pr(n_-)} + 1 + \tfrac{1}{18} M. \tag{4.12}$$

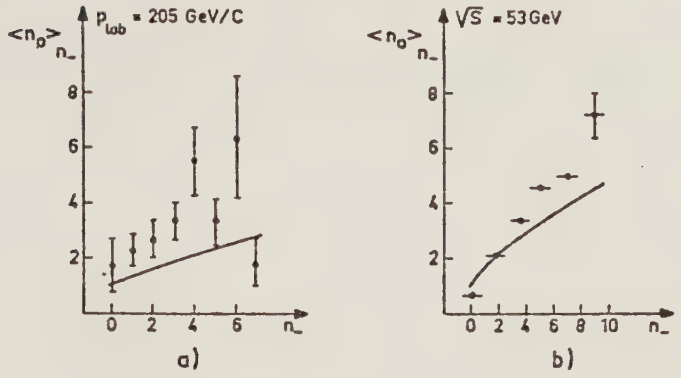

Fig. 2. The correlation $\langle n_o \rangle_{n_-}$ between π^o's and negative tracks at (a) 205 GeV/c and (b) 1500 GeV/c ($\sqrt{s}$ = 53 GeV), as predicted in our model.

Of course in this equation, $\Pr(n_-) = \Pr(n_c = 2n_- + 2)$ as given by eqs. (4.5) to (4.8). M is given by eq. (3.5), and 1 stands for the average number $\langle \pi^o \rangle_d$ of π^o's contained in the diffractive component. The last term in (4.12), which is independent of n_-, arises from the particular state 6 in table 1, which contains no negative particle. The result for the numerator N in the first term of eq. (4.12), is given in appendix B.

5. Discussion and conclusion

We want to discuss two points, one concerning the correlation functions, the other one, the energy dependence in our model.

(i) We have stressed the fact that in our model, all f_q's are linear functions of $\langle n_c \rangle$, with a slope which is either positive or zero (it is zero for q larger than the largest possible number of charged particles present in one fireball). This behaviour is in rough agreement with the data for f_2^- [9], f_2^c and f_3^c [7]. Our prediction for f_3^- is $f_3^- \simeq 0$, since we never consider pairs of resonances (like $A_2^+ A_2^-$ for example) yielding at least $3\pi^-$'s. The data presented for f_3^- [9] and f_4^c [7] do not agree with the above described behaviour. One should however keep in mind that the experimental determination of f_q for high values of q is extremely difficult, as each f_q appears as the result of large cancellations between the large values of the moments. As a proof of that fact, we recall that we obtain a perfect agreement with all values of the moments c_q (q = 2, 3, 4), at all energies (see table 2), with at the same time an f_4^c increasing with energy, whereas this coefficient is given with a large negative slope by Slattery [7]. At any rate, in view of the physical meaning which is to be attributed to the slope of f_q^c as a function of $\langle n_c \rangle$ (eq. (3.6)), we stress the importance of obtaining good numbers for them from experiment.

(ii) Consider eq. (3.5) which gives M, the average number of multiperipheral fireballs, as a function of $\langle n_c \rangle$, once B is fixed. Our numerical value for B (eq. (4.1) in the

SU(3) singlet model) is $B = 1.519$, so that M vanishes when $\langle n_c \rangle \simeq 5.0$. Therefore for energies smaller than $p_{\text{lab}} \simeq 50$ GeV/c our model clearly runs into trouble. We attribute this to our crude treatment of the diffractive mechanism which in the lower energy region becomes as important as the multiperipheral one. Thus, in this region, where the multiplicity is small, the rigorous constancy of B with energy and the independence of the two mechanisms may not be valid. On the other hand, our rather high value of B (eq. (4.1)), which is too large at $p_{\text{lab}} \simeq 50$ GeV/c, is obtained under the simplifying assumption that diffraction produces only uncorrelated negative particles, in an energy independent way. Concerning the energy dependence in our model, we would like to add the following comments. We have explained (subsect. 4.1) how the quasi-constancy of the c_q coefficients (quasi KNO scaling) results from a competition between fragmentation and pionization. In particular, eq. (4.1) indicates that if B increases with energy, so does the average number of particles per multiperipheral cluster. Thus we approach the point of view of Lévy [8] that the average number of particles per cluster increases with energy. It is difficult to resolve this question of energy dependence within the presently available experimental range. Considering only two extreme cases, one can ask the question: will the additional energy go into creating more fireballs decaying into a fixed number of particles, or will the average number of particles for a fixed number of fireballs increase? In general, of course, both the mass of each fireball and the number of particles it decays into can vary with energy. Various possibilities have been considered within the framework of the thermodynamical model in ref. [11], where the energy independent fireball mass appears as a limiting case. In our model, however, the structure of each fireball is specified and enables us to make predictions for correlations, multiplicities for different particles, n-particle cross sections, and compare these with experiment.

Because of these considerations, the results of the SU(3) singlet model for ultra high energies should be taken with caution. For example, in the present form of our model, each n-particle cross section tends to zero when the energy goes to infinity. This is characteristic of our point of view of the two-component picture (diffractive and multiperipheral components do *not* add), and happens because of the factor $\exp(-\langle Y_1 \rangle - \langle Y_2 \rangle)$ in eqs. (4.5) to (4.8), with $\langle Y_1 \rangle$, $\langle Y_2 \rangle$ being strictly proportional to $\langle n_c \rangle$ (assuming $\langle n_c \rangle$ continues to grow with energy and the denominator in eqs. (4.6) and (4.7) remains constant). The question of whether this fact ultimately tests the structure of the pomeron goes far beyond the ambitions of our present model. All we need in our multiperipheral chain is vacuum quantum number exchange. The exact nature of the pomeron singularity, which may be very complicated, is not required. For that reason too we do not think that the Finkelstein-Kajantie disease [12] is a serious problem, the more so since recent experimental results seem to indicate that the pomeron is not a simple factorisable pole with intercept 1. What we believe is more general than the way it is illustrated in our model, taken literally, is the important notion of competition between two mechanisms, which implies a strong connection between the energy dependence of B and the composition of fireballs.

We thank D. Levy for stimulating discussions and R. Balian for helpful comments concerning the question of correlations, H. Miettinen for discussions and M. Le Bellac for pointing out a numerical error in the original version of the manuscript.

Appendix A

A.1. General results

Let z be a positive integer random variable with probability $P(z)$. $P(z)$ is normalized to

$$\sum_{z=0}^{\infty} P(z) = 1 . \tag{A.1}$$

The first characteristic function $\varphi_1(t)$ is defined as

$$\varphi_1(t) = \langle e^{itz} \rangle = \sum_z e^{itz} P(z) . \tag{A.2}$$

The moments $\langle z^q \rangle$ of $P(z)$ are the coefficients of the Taylor expansion of φ_1:

$$\varphi_1(t) = \sum_q \frac{\langle z^q \rangle (it)^q}{q!} . \tag{A.3}$$

The second characteristic function $\varphi_2(t)$ is defined as

$$\varphi_2(t) = \log \varphi_1(t) . \tag{A.4}$$

A.2. Definition of the Mueller coefficients

The Mueller coefficients [5] are the coefficients of the expansion of φ_2 considered as a function of u defined by

$$e^{it} = 1 + u ,$$

$$\psi(u) = \varphi_2(-i \log(1+u)) = \sum_q f_q \frac{u^q}{q!} . \tag{A.5}$$

A.3. Physical meaning of the f_q coefficients

Let us compute f_q for a random variable $n = mz$, where z is a Poisson variable with $\langle z \rangle = a$, m a fixed number. One finds

$$\varphi_2(t) = -a + a\,e^{itm}\,, \tag{A.6}$$

so that

$$\psi(u) = -a + a(1+u)^m\,,$$

from which we derive

$$f_q = a\,m(m-1)\cdots(m-q+1)\,. \tag{A.7}$$

This is the basic formula we use all along in our paper. It means that the f_q correlation coefficients are linear in the average value ma of the random variable n. Moreover $f_q = 0$ if m is a positive integer and $q > m$. For $q < m$, the f_q's are all positive. Thus these coefficients are well adapted to the measurement of the number m of the considered objects which are contained in each of the z uncorrelated clusters.

A.4. Additivity of the f_q coefficients

It is well known that the second characteristic function φ_2 for the sum $x = y + z$ of two *independent* random variables is the sum of the two individual φ_2 functions

$$\varphi_2^x = \varphi_2^y + \varphi_2^z\,. \tag{A.8}$$

From this property we deduce that if, $x = \Sigma_i z_i$, with z_i independent from each other, then

$$f_q^{(x)} = \sum_i f_q^{(i)}\,. \tag{A.9}$$

Formulae (A.7) and (A.9) are the basic equations for deriving eq. (3.6) of our paper.

A.5. Relation between the f_q coefficients and the moments $\langle z^q\rangle$

The relations between the f_q's and the moments $\langle z^q\rangle$ are given by (cf. eqs. (A.3), (A.4) and (A.5))

$$\sum_q f_q \frac{u^q}{q!} \equiv \log\left\{\sum_p \frac{\langle z^p\rangle}{p!}\,(\log(1+u))^p\right\} \tag{A.10}$$

and the inverse relations by

$$\sum_p \frac{\langle z^p\rangle (it)^p}{p!} \equiv \exp\left\{\sum_q f_q \frac{(e^{it}-1)^q}{q!}\right\}. \tag{A.11}$$

Appendix B

We give here the formulae used in the calculation of $\langle \pi^0 \rangle_{n_-}$, the average number of π^0's for a given number of negative secondaries.

The expression for

$$N = \sum_{n_0} n_0 P_I(n_0, n_-)$$

in eq. (4.12) is given by

$$N = \exp(-\langle Y_1 \rangle - \langle Y_2 \rangle)(C_1 G_1(n^-, \langle Y_1 \rangle, \langle Y_2 \rangle) + C_2 G_2(n^-, \langle Y_1 \rangle, \langle Y_2 \rangle)) ,$$

where

$$C_1 = \tfrac{7}{9} M , \qquad C_2 = \tfrac{1}{8} M ,$$

and $\langle Y_1 \rangle$ and $\langle Y_2 \rangle$ are as in eq. (4.6).

For $n_- = 2p$, we obtain

$$G_1 = (4\langle Y_2 \rangle)^{p-1} \frac{(p-1)!}{(2p-1)!} \langle Y_1 \rangle L_{p-1}^{\frac{1}{2}} \left(-\frac{\langle Y_1 \rangle^2}{4\langle Y_2 \rangle} \right) ,$$

$$G_2 = (4\langle Y_2 \rangle)^{p-1} \frac{(p-1)!}{(2p-2)!} L_{p-1}^{-\frac{1}{2}} \left(-\frac{\langle Y_1 \rangle^2}{4\langle Y_2 \rangle} \right) .$$

For $n_- = 2p + 1$, we obtain

$$G_1 = (4\langle Y_2 \rangle)^{p} \frac{p!}{(2p)!} L_{p}^{-\frac{1}{2}} \left(-\frac{\langle Y_1 \rangle^2}{4\langle Y_2 \rangle} \right) ,$$

$$G_2 = (4\langle Y_2 \rangle)^{p-1} \frac{(p-1)!}{(2p-1)!} \langle Y_1 \rangle L_{p-1}^{\frac{1}{2}} \left(-\frac{\langle Y_1 \rangle^2}{4\langle Y_2 \rangle} \right) .$$

References

[1] Soviet-French Collaboration, contribution submitted to the 16th Int. Conf. on high energy physics, Batavia, 1972;
J.W. Chapman et al., Michigan-Rochester Collaboration, contribution submitted to the 16th Int. Conf. on high energy physics, Batavia, 1972;
G. Charlton et al., Phys. Rev. Letters 29 (1972) 515;
F.T. Dao et al., Phys. Rev. Letters 29 (1972) 1627;
G. Flugge et al., CERN report (August, 1972) submitted to the 16th Int. Conf. on high energy physics, Batavia, 1972.

[2] Z. Koba, H.B. Nielsen and P. Olesen, Nucl. Phys. B40 (1972) 317.
[3] F. Hayot and A. Morel, Saclay preprint DPh-T/73/4 (January, 1973).
[4] K. Fialkowski, Phys. Letters 41B (1972) 379;
H. Harari and E. Rabinovici, Phys. Letters B43 (1973) 49;
K. Fiałkowski and H.I. Miettinen, Phys. Letters B43 (1973) 64;
C. Quigg and J.D. Jackson, NAL preprint NAL-THY-93 (October, 1972).
[5] A.H. Mueller, Phys. Rev. D4 (1971) 150.
[6] G. Szego, Orthogonal polynomials, vol. 23 (American Mathematical Society Colloquium Publication, 1939).
[7] P. Slattery, University of Rochester preprint COO-3065-26 (1972).
[8] D. Lévy, Saclay preprint DPh-T/73/9 (February, 1973).
[9] M. Jacob, Report presented at the 16th Int. Conf. on high energy physics, Batavia, 1972.
[10] M. Antinucci et al., CERN preprint (January, 1973).
[11] A. Bassetto, G. Ranft and J. Ranft, Nuovo Cimento Letters 5 (1972) 841;
G. Ranft and J. Ranft, Nucl. Phys. B53 (1973) 217.
[12] J. Finkelstein and K. Kajantie, Nuovo Cimento 56 (1968) 659.

Note added in proof

Since this work was done, it has become clear from ISR experiments that among all high multiplicity events there exists a class of events where *all* the particles produced are fragments of one of the incoming protons that has been diffractively excited. This class of events populates only the low multiplicities and contributes to about 20% of the inelastic cross section. In our present work we do not take into account these purely diffractive events, in which no centrally produced particle occurs. They are excluded from consideration at the point of our text, where we assume that the probability of producing x particles multiperipherally and y particles as fragments of the incoming proton factorises (eq. (2.10) and eq. (2.11)). Our model therefore should be considered as describing only that large majority of events which are not purely diffractive.

REPRINT N°4

ANALYSIS OF TWO PARTICLE RAPIDITY CORRELATIONS AT ISR ENERGIES

Nuclear Physics B68 (1974) 323–330. North-Holland Publishing Company

ANALYSIS OF TWO PARTICLE RAPIDITY CORRELATIONS AT ISR ENERGIES

F. HAYOT AND A. MOREL
Service de Physique Théorique, Centre d'Etude Nucléaires de Saclay, BP. n° 2 — 91190 Gif-sur-Yvette

Received 12 July 1973

Abstract: The short range two particle rapidity correlations at ISR energies are interpreted in terms of clustering effects in the central rapidity region. It is shown that the experimental data give an indication that large clusters are produced, and as a consequence one should observe short range correlation between π^-'s. The energy dependence of the data is also simply explained.

A good description of the measured topological cross sections and, correspondingly, of the integrated correlation coefficients f_q has been obtained in the framework of the two component model [1]. In this kind of model, the major part of the energy dependence of the f_q's is interpreted as due to the existence of two competing mechanisms, diffraction and pionization. As a consequence the respective contributions of these two components to the correlations is somewhat difficult to isolate. The measurement at the ISR of the two particle rapidity correlation function

$$R(y_1, y_2) = \sigma_{\text{in}} \frac{\mathrm{d}^2\sigma}{\mathrm{d}y_1\,\mathrm{d}y_2} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}y_1}\frac{\mathrm{d}\sigma}{\mathrm{d}y_2} - 1, \tag{1}$$

in particular in the central region in rapidity, makes it possible to study the pionization component in a somewhat more independent way. In doing so, we take the point of view that independent clusters are emitted in the central region and propose to analyse the data for $R(y_1, y_2)$ in terms of the size and composition of these clusters. An extremely simple picture emerges from this analysis, which yields a very good description of the observed short range correlations in the central region.

The correlation function (1) has been measured by the CHV collaboration for one charged particle and one γ-ray at several energies [2]. Similar results for $R(y_1, y_2)$ as to its shape and normalization have been obtained by the Pisa-Stony Brook group for correlations between two charged particles [3]. The following features of the data emerge:

(i) When $y_1 = 0$ (y_1 is the rapidity of the charged particle) the correlation is of course symmetrical in y_2, maximum at $y_2 = 0$, and falls off rapidly. This is clearly a short range effect: the two particles tend to stick together.

(ii) When $y_1 = -2.5$, the correlation is still maximum at $y_1 \simeq y_2$, and the short

range effect is still present. However some long range effects show up. This long range effect gives a more or less flat contribution to $R(y_1, y_2)$ when $y_2 \gtrsim 0$, and when y_2 reaches the limits of the available rapidity range, it gives anticorrelation, the function $R(y_1, y_2)$ turning negative rather abruptly. The anticorrelation is more visible in the $\sqrt{s} = 30$ GeV data than in those at $\sqrt{s} = 53$ GeV, since in the latter case the available rapidity range is larger. This effect at the edges is expected. The correlation being positive over most of the rapidity interval, it has to become negative at the edges in order for the well-known sum rule to be verified [4].

(iii) When $y_1 = 0$ the short range effect appears to be independent of energy, whereas at $y_1 = -2.5$ the correlation seems to be increasing slowly with energy. In particular the short range maximum is rising.

The long range phenomenon can be related to the existence of the single diffractive dissociation events, associated with the proton, which have been recently observed [5]. In fact, when y_2 is at the edge of the available rapidity range and therefore very probably belongs to a diffractive event, the probability of finding at the same time a particle in the central region is small. This implies that $1/\sigma_{in}\, d^2\sigma/dy_1\, dy_2$ is smaller than the product $1/\sigma_{in}\, d\sigma/dy_1\ 1/\sigma_{in}\, d\sigma/dy_2$ and thus $R(y_1, y_2)$ is negative. On the contrary, when y_2 is in the central region, the probability of finding another particle in the same region is enhanced (even in the absence of any short range effect) relative to the corresponding one particle probability, since the event is not expected to be of diffractive type. Therefore $R(y_1, y_2)$ will be positive. A quantitative study of the long range correlation has been done in ref. [6]. We will now concentrate on the short range part of the correlation.

A first class of events in n-particle production is associated with diffractive dissociation. Let σ_d be the corresponding cross section. The non-diffractive inelastic cross section is thus defined as $\sigma_{nd} = \sigma_{in} - \sigma_d$. It corresponds to those events which contain at the same time particles that are produced centrally and particles that are fragments either of the projectile or target. We will now work in the framework of a model we have recently used to study n-particle cross sections [7]. In this model the centrally produced particles are decay products of uncorrelated clusters distributed along some multiperipheral chain. As a consequence, these clusters will have a rather wide rapidity spectrum, and we will obtain short correlations between two particles which are carried along within the same cluster. For the same reason, we expect only small correlations between two fragments since the corresponding clusters are, by definition, well localized around either the target or projectile rapidity.

First we compute the one particle distribution in the central region*. We consider that each cluster decays isotropically in its center of mass. In this frame, where the

* In what follows, we consider that all quantities have been averaged over the transverse momentum distribution, and we always identify the true rapidity $y = \frac{1}{2} \log (E+p_L)/(E-p_L)$ with the ISR variable $\eta = -\frac{1}{2} \log (p+p_L)/(p-p_L)$.

rapidity is z, the distribution is given by [8]

$$p(z) \simeq \frac{1}{2\,\mathrm{Ch}^2 z}.^{*} \tag{2}$$

Next we consider that the cluster rapidity distribution is essentially a plateau of width Δ, in the spirit of a multiperipheral production, and furthermore that clusters have small transverse momenta, so that they are practically at rest in the frame where their rapidity is zero. By averaging over the cluster rapidity distribution, we obtain for the one particle distribution in the total center of mass:

$$P^{c}(y) = \frac{1}{2\Delta}(\mathrm{th}(\tfrac{1}{2}\Delta + y + \mathrm{th}(\tfrac{1}{2}\Delta - y)). \tag{3}$$

Here $P^{c}(y)$ is normalized to unity. The superscript c refers to the centrally produced particles which we have discussed above. In our simple picture of the central region, the experimentally observed central plateau in y is obtained for $|y| < \frac{1}{2}\Delta$ as a reflection of the assumed plateau for the clusters. Eq. (3) is then supposed to reproduce the one particle distribution only in the central region.

We now turn to the two particle distribution. For two particles arising from two different, independent, clusters, the distribution is

$$V(y_1, y_2) = P^{c}(y_1)\, P^{c}(y_2), \tag{4}$$

with $P^{c}(y)$ as given by eq. (3).

When the two particles belong to the same cluster, the distribution is given by:

$$I^{c}(y_1, y_2) = \frac{1}{4\Delta}\int_{-\frac{1}{2}\Delta}^{\frac{1}{2}\Delta} \mathrm{d}Y\, \delta(Y + z_1 - y_1)\, \delta(Y + z_2 - y_2)\, \frac{\mathrm{d}z_1}{\mathrm{Ch}^2 z_1}\, \frac{\mathrm{d}z_2}{\mathrm{Ch}^2 z_2}.$$

The integral can be done explicitly, but its properties can easily be deduced from the form:

$$I^{c}(y_1, y_2) = \frac{1}{4\Delta}\int_{-\frac{1}{2}\Delta - y_1}^{\frac{1}{2}\Delta - y_1} \frac{\mathrm{d}u}{\mathrm{Ch}^2 u}\, \frac{1}{\mathrm{Ch}^2(u - (y_2 - y_1))}. \tag{5}$$

These properties will be discussed below.

The relative amount of the contributions (4) and (5) to the two particle cross section in the central region will depend on the particular cluster model one wants to consider. If the clusters, which follow a Poisson distribution are all taken to be of the same nature, we obtain for y_1 and y_2 in the central region:

$$\frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}^2\sigma^{c}}{\mathrm{d}y_1\,\mathrm{d}y_2} = \langle p\rangle^2 M^2 P^{c}(y_1)\, P^{c}(y_2) + \langle p(p-1)\rangle M I^{c}(y_1 y_2). \tag{6}$$

* This formula holds for sufficiently light particles, like π's, which constitute the majority of the produced particles.

Here M denotes the average number of clusters; p is the number of undistinguished particles in a cluster and the symbol $\langle\,\rangle$ refers to average values over the multiplicity distribution inside this cluster*. In the same way for the one particle cross section:

$$\frac{1}{\sigma_{nd}}\frac{d\sigma^c}{dy_1} = \langle p\rangle M P^c(y_1). \tag{7}$$

According to eqs. (6) and (7), the correlation function in the central region is:

$$R(y_1,y_2) = \frac{\sigma_{in}}{\sigma_{nd}}\frac{d\sigma^c}{dy_1}\frac{d\sigma^c}{dy_2}\Big/\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2} - 1 + \langle p(p-1)\rangle M I^c(y_1,y_2)\Big/\frac{1}{\sigma_{nd}}\frac{1}{\sigma_{in}}\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2}. \tag{8}$$

Here $1/\sigma_{in}\, d\sigma/dy$ represents the experimentally measured one particle distribution. The first term in (8) corresponds to a long range effect, and has been studied by Pirila and Pokorski [6].

Let us now concentrate on the effective short range term:

$$E^c(y_1,y_2) = \frac{\sigma_{nd}}{\sigma_{in}}\frac{\langle p(p-1)\rangle M I^c(y_1,y_2)}{\frac{1}{\sigma_{in}^2}\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2}}. \tag{9}$$

It is clear from (5) that whenever $|y_1| \ll \frac{1}{2}\Delta$, the function $I^c(y_1, y_2)$ depends essentially on the difference $y_1 - y_2$, falls off exponentially with a correlation length of order 1 [9], and is maximum when $y_1 = y_2$. Therefore we predict that when both y_1 and y_2 are well inside the experimentally obtained plateau ($P(y_1) = P(y_2) =$ constant) the short range correlation $E^c(y_1, y_2)$ remains the same, namely is maximum at $y_1 = y_2$, and for y_1 fixed depends only on $|y_1 - y_2|$. This is well supported by the CHV data [2] at $y_1 = 0$, and the Pisa-Stony Brook data [3] at $y_1 = 0$ and $y_1 = -0.85$.

We then turn to the question of the actual size of the short range correlations. Once the function $I^c(y_1, y_2)$ and the one particle inclusive cross section are known, eq. (9) shows that the measurement of the two-body cross section near $y_1 \simeq y_2$ can be considered as providing information about the composition and the size of the clusters. To be specific let us concentrate on the point $y_1 = y_2 = 0$, where things are easy to compute. Using eqs. (3), (5), (7) and (9), one obtains (in the limit $\mathrm{th}(\frac{1}{2}\Delta) \sim 1$)** for two undistinguished particles the relation

$$R(0,0) = \frac{\sigma_d}{\sigma_{nd}} + \frac{\langle p(p-1)\rangle}{\langle p\rangle}\frac{1}{3}\frac{1}{\frac{1}{\sigma_{in}}\frac{d\sigma}{dy}(0)}. \tag{10}$$

* $\langle p\rangle^2$ and $\langle p(p-1)\rangle$ have to be replaced by $\langle p_i\rangle\langle p_j\rangle$ and $\langle p_i p_j\rangle$ respectively in the case of different particles.

** This approximation is reasonable for $\Delta \gtrsim 3$. An estimate of Δ leads to $\Delta \gtrsim 3.5$ at ISR energies (see below).

$\langle p(p-1)\rangle/\langle p\rangle$ becomes $\langle pp_\gamma\rangle/\langle p_\gamma\rangle$ in the case of charged-neutral correlations. Experimentally, as a comparison between the Pisa-Stony Brook and CHV data shows the quantity $R(0,0)$ does not seem to differ substantially for correlations between two charged particles and one charged and one neutral particle.

Eq. (10) provides a good illustration of how independent cluster production shows up in the correlation function. In order to obtain an estimate of the quantity which refers to the cluster size, we take $R(0,0) = 0.65$ [2], $1/\sigma_{in}\, d\sigma/dy(0) = 1.8$*, and $\sigma_d/\sigma_{nd} = 20\%$ [1]. The magnitude of the last number is open to question and will be discussed later. With the preceding numbers, we obtain

$$\frac{\langle pp_\gamma\rangle}{\langle p_\gamma\rangle} \simeq \frac{\langle p(p-1)\rangle}{\langle p\rangle} \simeq 2.5. \tag{11}$$

The point we want to emphasize is that this number should be considered a large number. Note for example that an independent emission of ρ_0 or σ resonances leads to a value of one for the quantity (11)**. Moreover from (11) one obtains for two negative particles (if clusters are neutral):

$$\frac{\langle p_-(p_--1)\rangle}{\langle p_-\rangle} = \tfrac{1}{2}\left[\frac{\langle p(p-1)\rangle}{\langle p\rangle} - 1\right] \simeq 0.75. \tag{12}$$

This gives a value of 0.47 for $R(0,0)$ in eq. (10) i.e. the contribution of the short range component to $R(0,0)$ is as big as that given by the height of the long range plateau, taken to be 0.20. Therefore sizeable short correlations between $2\pi^-$'s are expected, contrary to what happens if independent production of single resonances occurs. Experimental results on correlations between two particles of the same charge will therefore provide important physical information.

We now proceed to discuss the y_1 and y_2 dependence of the function $R(y_1, y_2)$. We shall use the CHV data [2] which exist at different energies for $y_1 = 0$ and -2.5. We choose to make the comparison at the highest available energy, namely $\sqrt{s} = 53$ GeV, in order to minimize the importance of diffractive effects at the point $y_1 = -2.5$. $R(0,0)$ is fixed to its experimental value of 0.65 as explained before, i.e. by setting $\langle pp_\gamma\rangle/\langle p_\gamma\rangle = 2.5$ (eq. (11)), $\sigma_d/\sigma_{nd} = 0.2$ and $1/\sigma_{in}\, d\sigma/dy(0) = 1.8$. The only quantity that is now left undetermined is the width Δ of the cluster rapidity distribution. We determine Δ at $\sqrt{s} = 30$ GeV through fitting with formula (3) the one particle cross section at high multiplicities [3], where events with central particles are expected to dominate. We found $\Delta \simeq 3.5$ and consequently $\Delta \simeq 4.5$ at $\sqrt{s} = 53$ GeV since Δ grows like $\log s$ if scaling holds. The contribution to $R(y_1, y_2)$ of the short range corre-

* This value corresponds to the coefficient of the $\log(s)$ term in the expression for the number of charged particle n_c (cf. ref. [10]).

** In the type of model we have considered previously [7], where for instance a $\rho\rho$ pair is produced in an isospin zero state, one obtains a value of 2 for expression (11).

lation I^c, which is now known, is of the form

$$E^c(y_1, y_2) \propto \frac{I^c(y_1, y_2)}{\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2}}, \tag{13}$$

and the last thing we have to do is to put into the denominator the shapes in rapidity of the one particle inclusive cross sections. In our model, these shapes are the same for a charged particle and a γ ray, for not too large $|y|$'s, and given by eq. (3) in the central region, the size of which is $\sim \Delta$. In order to present our results also outside this central region we use a numerical fit to the Pisa-Stony Brook distribution [3] at $\sqrt{s} = 30$ GeV, which coincides with $P^c(y)$ ($\Delta = 3.5$) for $|y| \lesssim 1$. To reconstruct the one particle distribution at $\sqrt{s} = 53$ GeV we use the scaling hypothesis. Accordingly, the one particle distribution at $\sqrt{s} = 53$ GeV and positive (negative) rapidity y_0 is equal to that one at $\sqrt{s} = 30$ GeV and rapidity $y_0 + 0.5$). We are now able to compute the short range contribution to $R(y_1, y_2)$, namely the quantity $E^c(y_1, y_2)$ normalized as explained above to the experimental value at $y_1 = y_2 = 0$. In fig. 1 we compare this quantity with

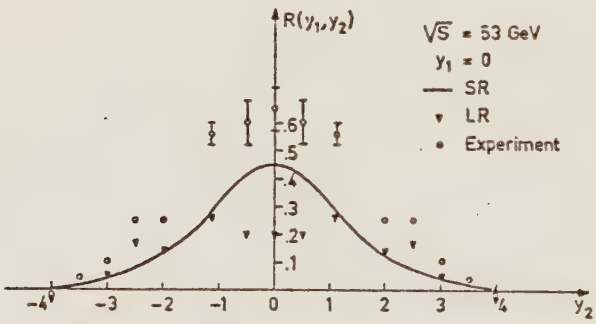

Fig. 1. The correlation function $R(y_1, y_2)$ at $\sqrt{s} = 53$ GeV and $y_1 = 0$. The experimental results [2] are shown as open circles. The curve SR corresponds to our prediction for the short range. The long range effect appears as black triangles. It is calculated as the difference between the experimental points and the calculated short range correlation.

the experimental data at $\sqrt{s} = 53$ GeV and $y_1 = 0$. The function $E^c(0, y_2)$ falls faster than the experimental distribution $R(0, y_2)$ for small values of y_2, and is of course symmetrical in y_2. The difference between the experimental $R(0, y_2)$ and $E^c(0, y_2)$ is the long range effect which is also shown in fig. 1. At $y_1 = y_2 = 0$ its value is 20% resulting from our choice for σ_d/σ_{nd}. It seems however to exhibit a rather large plateau of half-width of about 1.5, and then drops down towards the edges where it becomes finally negative the same way as $R(0, y_2)$ does, since at the edges the short range correlation is negligible. In fig. 2 we compare the short range correlation $E^c(y_1, y_2)$ with the data at $y_1 = -2.5$ and $\sqrt{s} = 53$ GeV. Our prediction is correct, namely $E^c(y_1, y_2)$ has a maximum around $y_2 = y_1$ and its value there is less than at $y_1 = y_2 = 0$. The data are not really precise, for instance the actual maximum of $R(y_1, y_2)$ is not well located,

and there is a lack of experimental points around $y_2 = 0$. For that reason we have not extracted the long range effect as previously. The shape of this long range correlation is a crucial point. Notice that, anyway, the long range part of the correlation which has to be added to $E^c(y_1, y_2)$ is no longer equal to σ_d/σ_{nd} because at $y_1 = -2.5$ the diffractive component should probably be taken into account.

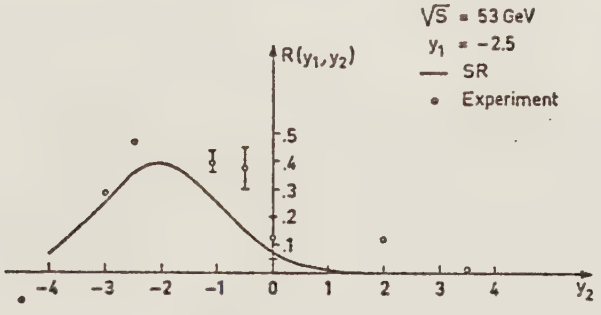

Fig. 2. The correlation function $R(y_1, y_2)$ at $\sqrt{s} = 53$ GeV and $y_1 = -2.5$. The experimental data [2] are shown as open circles. The curve SR corresponds to our prediction for the short range.

Let us finally discuss the energy dependence of the correlations. The CHV [2] data are compatible with $R(0,0)$ being constant with energy over the whole ISR energy range. With $(1/\sigma_{in})\, d\sigma/dy|_{y=0}$ also constant (scaling), we deduce that data favour energy independent clusters, namely that $\langle p(p-1)\rangle/\langle p\rangle$ or $\langle pp_\gamma\rangle/\langle p\rangle_\gamma$ are constants (see eq. (10)). This corresponds to the kind of model for clusters considered in ref. [7]. In this case scaling for the one particle distribution means that Δ/M is a constant and clearly $E^c(y_1, y_2)$ does not depend on energy and is the same whenever both y_1 and y_2 are inside the central plateau region. In particular, its maximum value at $y_1 = y_2$ is y_1 independent. This seems to be borne out by the data. It is moreover clear that for any value of y_1 fixed initially at a given energy outside the central region there will exist a sufficiently high energy where this value of y_1 becomes central, since the plateau extends logarithmically with energy if scaling holds. Therefore any correlation at a given y_1 outside the central region will tend towards $R(0, y_2)$; in particular its maximum will have to increase with energy towards $R(0,0)$. Good data are needed to check this trend.

Let us conclude with a few remarks. First we want to stress the importance of getting information about the diffractive component directly from experiment, and in particular about the corresponding inclusive rapidity cross section, and the total diffractive cross section. They are necessary to obtain a better separation between the two components. Nevertheless, and this is our second remark, the fact that, in case short range correlations actually result from independent cluster production, the clusters have to be large, is independent of the particular value taken for σ_d/σ_{nd}. Even with $\sigma_d/\sigma_{nd} = \frac{1}{3}$, which is the rather large value chosen by Pirila and Pokorski [6], the two quantities $\langle p(p-1)\rangle/\langle p\rangle$ (for two charged particles) and $\langle pp_\gamma\rangle/\langle p_\gamma\rangle$ are still equal to 2 (eq. (10)).

In this respect we recall our prediction that sizeable short range correlations should be observed between two negative particles. With $\sigma_d/\sigma_{nd} = \frac{1}{3}$ we still estimate the amount of correlation due to the short range effect to be at least 0.16. It has to be noticed that in such a case a fit to $R(y_1, y_2)$ of the form exp. $[-|y_1 - y_2|/\xi]$ would be misleading. The correlation length ξ would appear to be very large, although it is only of order 1 [9] for the short range effect.

References

[1] K. Fiałkowski and H.I. Miettinen, Phys. Letters 43B (1973) 64;
H. Harari and E. Rabinovici, Phys. Letters 43B (1973) 49;
J.D. Jackson and C. Quigg, NAL preprint NAL-THY-92 (1972).
[2] H. Dibon et al., Phys. Letters 44B (1973) 313.
[3] Pisa-Stony Brook Collaboration, Contribution to the 16th Int. Conf. on high energy physics, Batavia, 1972.
[4] E. Predazzi and G. Veneziano, Nuovo Cimento Letters 2 (1971) 749.
[5] M.G. Albrow et al., Phys. Letters 44B (1973) 207.
[6] P. Pirila and S. Pokorski, Phys. Letters 43B (1973) 502.
[7] F. Hayot and A. Morel, Nucl. Physics B59 (1973) 493.
[8] E.L. Berger and A. Krzywicki, Phys. Letters 36B (1971) 380.
[9] A. Białas, K. Fiałkowski and K. Zalewski, Krakow preprint TPJU-6/73.
[10] M. Antinucci et al., Nuovo Cimento Letters 6 (1973) 121.

REPRINT N°5

WHAT CAN WE LEARN FROM SEMI-INCLUSIVE RAPIDITY CORRELATIONS ?

Nuclear Physics B86 (1975) 333–345. North-Holland Publishing Company

WHAT CAN WE LEARN FROM SEMI-INCLUSIVE RAPIDITY CORRELATIONS?

F. HAYOT * and M. LE BELLAC **
Department of Physics, University of Michigan †

Received 24 May 1974
(Revised 22 July 1974)

Abstract: We study a general formulation of semi-inclusive two-particle rapidity correlations for short-range models. We use it to compare with the 205 GeV NAL Bubble Chamber data different decay distributions for independently emitted clusters. We also comment on non-independent cluster production and on semi-inclusive correlations between charged and neutral particles.

1. Introduction

In this work we attempt to answer the following question: what kind of information can be obtained from the study of semi-inclusive rapidity correlations [1, 2]? A first advantage of semi-inclusive correlations has already been pointed out by Berger [1]: if one restricts oneself to multiplicities larger than $\sim \frac{1}{2}\bar{n}$, it is possible to ignore diffractive effects (in a two-component type of model). This is certainly a very interesting point since the way to handle diffraction is far from clear. We will therefore concentrate in this work on the non-diffractive component of the production mechanism.

Our first goal is to derive a general formula for semi-inclusive correlations, valid both in multiperipheral (MP) and in cluster type models. By cluster models we mean, as is now fashionable [3], production of independent neutral clusters which decay into the observed particles. In sect. 2 we derive in a simple and physically transparent way a formula first obtained by Arnold and Thomas [4], which allows us to calculate semi-inclusive correlations from inclusive ones. We demonstrate explicitly the equivalence between MP and cluster models which has been recently questioned [5].

In sect. 3 we perform a phenomenological analysis of NAL [6] and ISR [7] data, using simple versions of the cluster model. In particular we compare various distributions of charged particles within the clusters, and obtain good agreement with the

* Permanent address: Service de Physique Théorique CEN Saclay.
** Permanent address: Faculté des Sciences, Université de Nice.
† Work supported by the US Atomic Energy Commission.

NAL data, and also with the 62 GeV data of the Pisa-Stony Brook experiment.

In sect. 4 we comment about neutral charged correlations from the point of view of independent cluster models. Finally, in sect. 5, we look at the case of non-independent clusters. This section is rather technical. However, we point out the important fact that the combined knowledge of topological cross sections and semi-inclusive correlations gives a clue as to the cluster multiplicity distribution. Our most important theoretical and phenomenological points are summarized in sect. 6.

2. Semi-inclusive correlations from inclusive ones

Let us first derive a general formula for semi-inclusive correlations, which depends only on the short range nature of the correlations, namely on the fact that the width of the multiplicity distribution $P(n)$ is of order $\overline{(n)}^{\frac{1}{2}}$. We define as usual the semi-inclusive and inclusive one and two particle distributions:

$$\rho_n(y) = \frac{1}{\sigma_n}\frac{\mathrm{d}\sigma_n}{\mathrm{d}y}\,, \qquad \rho(y) = \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}y}\,,$$

$$C_n(y_1,y_2) = \frac{1}{\sigma_n}\frac{\mathrm{d}^2\sigma_n}{\mathrm{d}y_1\mathrm{d}y_2} - \rho_n(y_1)\,\rho_n(y_2)\,,$$

$$C(y_1,y_2) = \frac{1}{\sigma}\frac{\mathrm{d}^2\sigma}{\mathrm{d}y_1\mathrm{d}y_2} - \rho(y_1)\,\rho(y_2)\,.$$

From the obvious relation

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}y_1\mathrm{d}y_2} = \sum_n \frac{\mathrm{d}^2\sigma_n}{\mathrm{d}y_1\mathrm{d}y_2}\,,$$

we derive the following general expression for $C(y_1,y_2)$:

$$C(y_1,y_2) = \sum_n P(n)\,C_n(y_1,y_2) + \sum_n P(n)\,(\rho_n(y_1)-\rho(y_1))\,(\rho_n(y_2)-\rho(y_2))\,. \tag{1}$$

At ultra-high energy, only the first term in (1) will survive, since the second term is of the order of $1/\overline{n}$, as we explain in more detail below. Moreover, since the correlations are assumed to be short range, we can replace

$$\sum P(n)\,C_n$$

by $C_{\overline{n}}$, with an error of the order of $1/\overline{n}$, so that

$$C(y_1,y_2) = C_{\overline{n}}(y_1,y_2) + O(1/\overline{n})\,. \tag{2a}$$

In order to generalize this result to arbitrary values of n, we modify the multiplicity distribution in the following way:

$$P(n) \to P_z(n) = \frac{z^n P(n)}{\sum z^n P(n)} . \tag{3}$$

In the case of the MP model, z is to be interpreted as the square of the coupling constant and by convention the physical value corresponds to $z = 1$. For cluster models eq. (3) has only a formal meaning, but in each case we can define z-independent one and two-particle densities, as well as z-dependent moments $\bar{n}(z)$ and $f_2(z)$. If we choose z in such a way that n is precisely equal to $\bar{n}(z)$ *, we can generalize (2a) to:

$$C(y_1, y_2, z) = C_n(y_1, y_2, n = \bar{n}(z)) + O(1/n) . \tag{2b}$$

We have checked numerically, for example in the case of cluster models with ρ-ρ clusters (see sect. 3), that the $O(1/n)$ error in (2b) is typically of a few percent and never exceeds 15% even for low values of n and low energies. In the case of the cluster model considered in ref. [1] our result is even exact.

At finite (NAL and ISR) energies, we have to take into account the second term in eq. (1). If we assume that $\rho(y)$ and $\rho_n(y)$ have each a plateau of length Y

$$\rho(y) = \frac{\bar{n}}{Y} , \qquad \rho_n(y) = \frac{n}{Y} ,$$

the second term in (1) is found to be equal to $(\bar{n} + f_2)/Y^2$, where f_2 is the second Mueller's moment. Generalizing to arbitrary z we finally get our basic equation (first derived by Arnold and Thomas [4])

$$C_n(y_1, y_2; n = \bar{n}(z)) = C(y_1, y_2, z) - \frac{1}{Y^2}(\bar{n}(z) + f_2(z)) . \tag{4}$$

Before turning to specific applications let us make a few comments about (4):

(i) One sees from eq. (4) that the semi-inclusive correlation is given by a function depending on $y_1 - y_2$ sitting on a constant negative background. This background is of the order of $1/Y$ and thus disappears at ultra-high energy, but is quantitatively important at present energies. Notice that the background term is essential in order for the sum rule

$$\int dy_1\, dy_2\, C_n(y_1, y_2) = -n$$

to be satisfied. In fact integrating (4) over (y_1, y_2) gives for the right-hand side $f_2(z) - (\bar{n}(z) + f_2(z)) = -n$.

(ii) Experimentally it is found that at NAL energies $C_n(0, 0)$ is close to zero and therefore that there exists at those energies an almost complete cancellation between the two terms on the right-hand side of eq. (4). Thus there is some doubt whether terms of order $1/Y$ in C_n can be neglected as argued above before point (i). This seems a difficulty unavoidable in all short-range order approaches to semi-inclusive correla-

* In the steepest descent method used by Arnold and Thomas [4] to derive eq. (4), the value of z such that $\bar{n}(z) = n$ corresponds to the position of the saddle point.

tions at NAL and ISR energies. Indeed, as emphasized in the following section, it would be very important to separate experimentally the two terms in (4) which differ by their $y_1 - y_2$ dependence, in order to extract valuable information from the data.

(iii) Eq. (4) applies to any short range correlation model. The MP model does not specify the form of the functions $\bar{n}(z)$ and $C(y_1, y_2, z)$. The cluster model is slightly more restrictive: first the correlation length is z-independent and second, as is clear from sect. 3, $|C_n/n|$ is a non-decreasing function of n. This last behavior may turn out to be different in specific versions of the MP model, but there does not seem to be any fundamental difference between the MP and cluster models as far as semi-inclusive correlations are concerned, contrary to what has been asserted by Caneschi [5].

3. Independent cluster models

We now apply eq. (4) to the cluster model. We call $w(p)$ the probability that a given cluster decays into p observed (charged, or negative) particles, and $\lambda(z)$ the generating function for the cluster decay:

$$\lambda(z) = \sum_p z^p w(p) \ .$$

Since the clusters are assumed to be independent and neutral, the generating function for the total multiplicity distribution $G(z)$ is given by [8, 9]

$$G(z) = \exp\{\bar{k}(\lambda(z) - 1)\} \ ,$$

where $\bar{k}$ is the average number of clusters. We then have:

$$\bar{n}(z) = \bar{k} z \lambda'(z) = \bar{k} \langle p \rangle_z \ , \tag{5a}$$

$$f_2(z) = \bar{k} z^2 \lambda''(z) = \bar{k} \langle p(p-1) \rangle_z \ , \tag{5b}$$

and the inclusive correlation has the form: [10]:

$$C(y_1, y_2) = \bar{k} \langle p(p-1) \rangle_z P^{(2)}(y_1, y_2) \ ,$$

where $P^{(2)}$ is normalized to unity. In order to be definite, let us write the formulae in the case of $y_1 = 0$, assuming that the rapidity distribution in the cluster rest frame is a Gaussian of width δ, with $\delta \approx 0.7$. Then [11]

$$P^{(2)}(0, y) = \frac{1}{2\delta\sqrt{\pi}\, Y} \, e^{-y^2/4\delta^2} \ ,$$

and we find *:

* In MP models the Gaussian in y should be replaced by an exponential, and the correlation length may depend on z.

$$C_n(0,y) = -\frac{1}{Y^2}\,(n+\bar{k}\,\langle p(p-1)\rangle_z) + \frac{\bar{k}\,\langle p(p-1)\rangle_z}{2\delta\sqrt{\pi}\,Y}\,e^{-Y^2/4\delta^2} \quad . \tag{6}$$

Let us recall that z is to be determined by the equation $\bar{k}z\,\lambda'(z) = n$. In order to compare (6) with experiment, it is more convenient to work with the ratio R_n, defined as usual by:

$$R_n(y_1,y_2) = \frac{C_n(y_1,y_2)}{\rho_n(y_1)\,\rho_n(y_2)} \quad .$$

Using (6) we find for R_n:

$$R_n(0,y) = -\frac{1}{n}\left(1+\frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z}\right) + \frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z}\,\frac{Y}{n}\,\frac{e^{-y^2/4\delta^2}}{2\delta\sqrt{\pi}} \quad . \tag{7}$$

Eq. (7) is the basic formula which we will compare with the experimental data. It is valid for the class of models where neutral clusters are produced independently. The particular case where each cluster decays into a fixed number of particles has been treated in ref. [1]. The most important point about eq. (7) is that R_n(or C_n/n) depends only on the ratio $\langle p(p-1)\rangle_z/\langle p\rangle_z$, and it would be extremely interesting to extract this ratio from the experimental data.

As suggested by Berger [1], one can fit $R_n(0,y)$ with the formula [12]:

$$R_n(0,y) = -\frac{A_n}{n} + B_n\,\frac{e^{-y^2/4\delta^2}}{\rho_n(0)} \quad .$$

where, by comparison with eq. (7), the coefficients A_n and B_n are given by:

$$A_n = 1 + \frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z} \quad , \tag{8a}$$

$$B_n = \frac{1}{2\delta\sqrt{\pi}}\,\frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z} \quad . \tag{8b}$$

One can then determine from the data the ratio $\langle p(p-1)\rangle_z/\langle p\rangle_z$ for each multiplicity. Since in inclusive correlations, this ratio is only measured at $z = 1$, it is clear that semi-inclusive correlations provide some important information on cluster decay.

We choose to compare three different cases of cluster decay. The first one corresponds to the decay into a fixed number, p_0, of particles * (for definiteness, we consider the decay into negative particles). This case has already been considered in ref. [1] and corresponds to

$$\frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z} = p_0 - 1 \quad , \tag{9}$$

* More generally one can take:

$$w(p) = 1-a+a\,\delta(p-p_0) \quad ,$$

where $(1-a)$ is the probability that the clusters decay into neutral particles only.

independently of z. The second case is that of the $\rho - \rho$ model [13]. Each cluster decays into $(2\pi^+, 2\pi^-)$ with a probability $\frac{1}{3}$ and into $(\pi^+, \pi^-, 2\pi^0)$ with a probability $\frac{2}{3}$. One has:

$$z = \tfrac{1}{2}(-1 + \sqrt{1 + 8n/\bar{n}}) ,$$

$$\frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z} = \frac{z}{z+1} . \tag{10}$$

Finally we consider that $w(p)$ is a Poisson distribution for $p \geqslant 1$ with parameter p_0, while $w(0) = 0$ *. The average number of particles is then given by $\langle p\rangle = p_0(1 - e^{-p_0})^{-1}$, and

$$\frac{\langle p(p-1)\rangle_z}{\langle p\rangle_z} = p_0 z , \tag{11}$$

where z is solution of the equation:

$$p_0 z\, e^{p_0 z} = \frac{n}{\bar{k}} (e^{p_0} - 1) .$$

In order to make a useful comparison we require that all three models give identical inclusive correlations, i.e., $\langle p(p-1)\rangle_z / \langle p\rangle_z$ is the same at $z = 1$. We then have $p_0 = \frac{3}{2}$ (mathematical example only!) in (9) and $p_0 = \frac{1}{2}(\langle p\rangle = 1.27)$ in (11).

In table 1 we give the results of this comparison with the following values of the parameters:

$$\bar{k} = \tfrac{15}{4} , \qquad Y = 6 ,$$

corresponding to the top ISR energy, with an average negative multiplicity of 5. It can be seen from table 1 that the ratio $\langle p(p-1)\rangle_z / \langle p\rangle_z$, which is constant in the first case, increases with n in the $\rho\rho$ and Poisson models. This is in fact a general property of cluster models: using the fact that $\lambda(z)$ is a polynomial in z with positive coefficients, it is easy to show that $\langle p(p-1)\rangle_z / \langle p\rangle_z$ is always an increasing function of z, and thus of n.

Table 1 illustrates the importance of extracting from the data the ratio $\langle p(p-1)\rangle_z / \langle p\rangle_z$, since it appears that this quantity is rather sensitive to the properties of cluster decay, whereas $R_n(0, 0)$ is much less so, because of cancellations arising from the presence of the $1/n$ term in eq. (7).

We now consider the experimental data which are available from a 205 GeV NAL bubble chamber experiment [6], and also from the Pisa-Stony Brook experiment [7] at $\sqrt{s} = 23$ and 62 GeV. The lower energy ISR and the NAL data, which are at the same center-of-mass energy, are inconsistent with each other. We have chosen to

* $w(0)$ is the probability for the cluster to decay into neutral particles only. Hence our choice of $w(0)$ is somewhat arbitrary.

Table 1
$\langle p(p-1)\rangle/\langle p\rangle$ and $R_n^{cc}(0,0)$ as a function of n_c for the three cluster decay models under consideration.

	n_c	8	12	16	20	24
$w(p) = \delta(p-p_0)$	R_n^{cc}	0.31	0.18	0.13	0.10	0.08
$\rho\rho$ model	$\frac{\langle p(p-1)\rangle}{\langle p\rangle}$	1.84	2	2.12	2.18	2.24
	R_n^{cc}	0.27	0.18	0.14	0.12	0.10
Poisson dist.	$\frac{\langle p(p-1)\rangle}{\langle p\rangle}$	1.70	2	2.24	2.36	2.62
	R_n^{cc}	0.24	0.18	0.16	0.13	0.12

In the case $w(p) = \delta(p-p_0)$ the ratio $\langle p(p-1)\rangle/\langle p\rangle = 2$ independently of n_c. The value of n chosen to evaluate R_n from eq. (7) was $n = n_c - 2$. The choice of the parameters $\bar{k}$ and Y corresponds to the top ISR energy. Our comparison is made here for charged particles, in order to compare our values for R_n^{cc} with the ISR results.

compare the models with the NAL results, which are given in terms of rapidity for each well defined multiplicity.

The parameter Y is evaluated using the equation

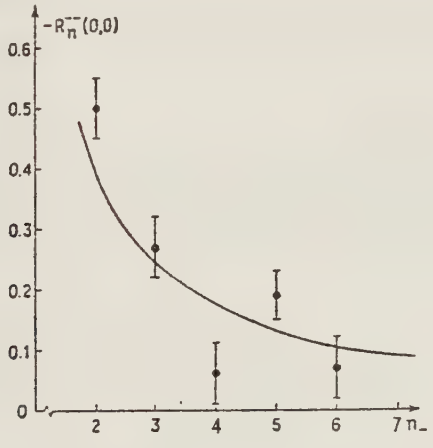

Fig. 1. A plot of $-R_n^{--}(0,0)$ versus n_-, where $R_n^{--}(0,0)$ is the semi-inclusive correlation function for two negatively charged particles with zero rapidities. We compare the 205 GeV NAL data with the following models for cluster decay: (i) fixed number of negative pions; (ii) ρ-ρ model; (iii) Poisson distribution (see sect. 3 for more details). As the three models give practically identical results, we have drawn only one curve.

$$Y = \frac{\bar{n}}{(1/\sigma)\,\mathrm{d}\sigma/\mathrm{d}y} \approx 3.85 \ ,$$

while $\bar{k} = \frac{3}{4}\bar{n}_-$ is determined using the "non-diffractive" value [14] $\bar{n}_- = 3.3$ of $\bar{n}_-$. It is seen from fig. 1 that $R_n^{--}(0, 0)$ is satisfactorily described by the models. We have also calculated the correlation for charged particles $R_n^{cc}(0, 0)$ and find that it is essentially zero for all n (more precisely $R_n^{cc}(0, 0)$ lies between 0 and 0.02), in agreement with the data. The limited statistics and the small length of the rapidity plot do not allow to make an analysis of the y dependence.

It turns out that the models also give a satisfactory description of the $\sqrt{s} = 62$ GeV Pisa-Stony Brook data (see table 1), but the disagreement of the NAL and ISR results at $\sqrt{s} = 23$ GeV must be explained before one can draw any conclusions. However, a complete analysis at the top ISR energy would be extremely useful since one may hope to separate the two terms in R_n (eq. (7)).

4. Correlations between charged and neutral particles

It would be interesting to have some information, not only on the distribution of charged particles, but also on the distribution of neutral particles within a cluster. To analyze this problem, it is useful to introduce the generating function $\Lambda(z, z_0)$ for cluster decay into negative and neutral pions (9):

$$\Lambda(z, z_0) = \sum_{p, p_0} z^p z^{p_0} w(p, p_0) \ . \tag{12}$$

In eq. (12), $w(p, p_0)$ is the probability for a cluster to decay into p negative and p_0 neutral pions. We will also need the derivatives of $\Lambda(z, z_0)$:

$$\Lambda^{(m,n)}(z, z_0) = \frac{\partial^{m+n}}{\partial z^m \partial z_0^n} \Lambda(z, z_0) \ , \tag{13}$$

Notice finally that $\lambda(z) = \Lambda(z, 1)$. We will first show that in the case of independent cluster production, the average number $\overline{n_0(n_-)}$ of π^0's as a function of n_- must be an increasing function of n_-. The general formula for $\overline{n_0(n_-)}$ is:

$$\overline{n_0(n_-)} = \bar{k}\, \Lambda^{(0,1)}(z, 1) \ , \tag{14}$$

where z is again given by eq. (5a). Since $\Lambda(z, z_0)$, is a polynomial in z and z_0 with positive coefficients, z must be an increasing function of n_-, and $\overline{n_0(n_-)}$ must be an increasing function of z. In the case of the ρ-ρ model one obtains:

$$\overline{n_0(n_-)} = \tfrac{4}{3}\bar{k}\, z = \tfrac{2}{3}\bar{k}\,(-1 + \sqrt{1 + 8n_-/\bar{n}_-}) \tag{15}$$

This expression gives an excellent fit of $\overline{n_0(n_-)}$ for $n_- \leqslant 6$ at 205 GeV [15] (fig. 2) but fails to reproduce the decrease of $\overline{n_0(n_-)}$ when n_- is larger than 6. We believe that this is an indication that for neutral clusters the cluster multiplicity distribution is not Poisson, as will be discussed in more detail in the next section.

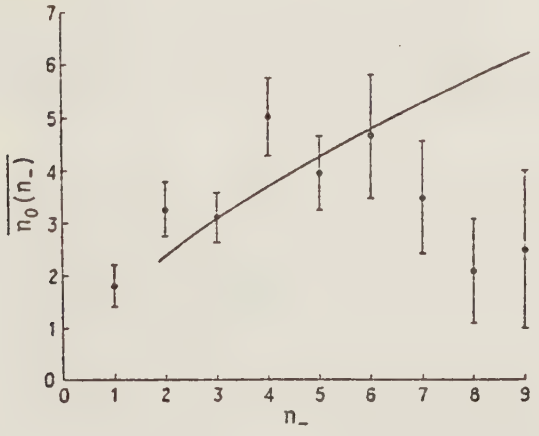

Fig. 2. Comparison of the ρ-ρ model with the 205 GeV data on $\overline{n_0(n_-)}$, the average number of π^0's produced for a given number of π^-'s, in the case of independent clusters.

We would also like to suggest that the n-dependence of the rapidity correlations between a negative (or charged) particle and a π^0 (in practice a γ-ray) at a fixed number of prongs can give some useful information on the distribution of neutral particles in a cluster. Indeed, one can easily prove the following equation:

$$\frac{1}{n_-\sigma_{n_-}}\frac{\mathrm{d}^2\sigma_{n-}}{\mathrm{d}y_-\,\mathrm{d}y_0}=\frac{\overline{n_0(n_-)}}{Y^2}+\frac{\Lambda^{(1,1)}(z,1)}{\lambda'(z)}\left(P^{(2)}(y_-,y_0)-\frac{1}{Y^2}\right), \tag{16}$$

where y_- and y_0 are the rapidity of the negative and neutral particles respectively. The ratio

$$\Lambda^{(1,1)}(z,1)/\lambda'(z)$$

is to be interpreted as the z-dependent average of pp_0 divided by $\langle p\rangle$:

$$\frac{\Lambda^{(1,1)}(z,1)}{\lambda'(z)}=\frac{\langle p\,p_0\rangle_z}{\langle p\rangle_z}.$$

The inclusive correlation between negative and neutral pions measures this ratio at $z=1$, but the semi-inclusive correlations (16) allow to study its variation with z.

In order to illustrate eq. (16), we briefly discuss 3 examples:

(i) No correlations between negative and neutral pions within a cluster. Then:

$$\langle p\,p_0\rangle_z/\langle p\rangle_z=\langle p_0\rangle$$

independently of z.

(ii) The cluster decays either into charged particles only, or into neutral particles only. Then,

$$\langle p\,p_0\rangle_z/\langle p\rangle_z=0\ .$$

(iii) The ρ-ρ model. Here

$$\langle pp_0\rangle_z/\langle p\rangle_z = \frac{2}{z+1} \ .$$

5. Production of non-independent clusters

If the clusters are not independent, i.e., if their multiplicity distribution $Q(k)$ does not follow a Poisson law, it is still possible to derive eq. (7) for R_n. Although, superficially, it seems that only quantities related to cluster decay appear in R_n, this is not true since the relation between n and z [(eq. (5a)] depends on the shape of $Q(k)$, and therefore on the cluster production mechanism.

Whereas for independent cluster emission only the average number of clusters, a quantity independent of z, appears in eq. (5a), such is not the case for non-independent clusters. It then becomes difficult to disentangle the contributions of production and decay mechanisms to semi-inclusive correlations.

However, we believe that it is still useful to determine experimentally the ratio

$$r(n) = \frac{\langle p(p-1)\rangle_n}{\langle p\rangle_n} \ ,$$

by measuring the coefficients A_n and B_n in relation (8), provided the multiplicity distribution $P(n)$ is available. Indeed, if we consider some precise form for $w(p)$ and assume we know $P(n)$, it should be possible, at least in principle, to calculate the cluster multiplicity distribution $Q(k)$ and then $r(n)$. Therefore, one ought to be able to express $r(n)$ directly as a function of $P(n)$ and to check whether the predicted form agrees with the experimental data on $r(n)$.

Let now $G(\lambda)$ be the generating function for cluster production:

$$G(\lambda) = \sum \lambda^k Q(k) \ .$$

$P(n)$ is then given by:

$$\begin{aligned} P(n) &= \frac{1}{n!}\frac{\partial^n}{\partial z^n} G(\lambda(z))|_{z=0} \\ &= \frac{1}{n!}\frac{\partial^{n-1}}{\partial z^{n-1}} \lambda'(z)\, G'(\lambda(z))|_{z=0} \ . \end{aligned} \tag{17}$$

On the other hand, it is easily shown, using for example, the formalism of ref. [1], that

$$P_1(n) \equiv \langle p(p-1)\rangle_n = \frac{1}{n!}\frac{\partial^n}{\partial z^n} z^2 \lambda''(z)\, G'(\lambda(z))|_{z=0} \ , \tag{18}$$

$$\langle p \rangle_n = \frac{1}{n!} \frac{\partial^n}{\partial z^n} z\lambda'(z)\, G'(\lambda(z))|_{z=0} = n\, P(n)\,.$$

Then, replacing $\lambda(z)$ by its expression in terms of $w(p)$, $P(n)$ and $P_1(n)$ can be re-written as:

$$P(n) = \sum_{p=1}^{n} \frac{p w(p)}{n(n-p)!} \frac{\partial^{n-p}}{\partial z^{n-p}} G'(\lambda(z))|_{z=0}\,,$$

$$P_1(n) = \sum_{p=2}^{n} \frac{p(p-1)\, w(p)}{(n-p)!} \frac{\partial^{n-p}}{\partial z^{n-p}} G'(\lambda(z))|_{z=0}\,.$$

Let us now study two relevant examples:

(i) Each cluster decays into either one or two negative particles, as is the case for the $\rho\rho$ model.

Here $\lambda(z) = zw(1) + z^2 w(2)$, and $P_1(n)$ is determined by the recursion formula:

$$P_1(n) = \frac{2w(2)}{w(1)} ((n-1)\, P(n-1) - P_1(n-1))\,. \tag{19}$$

In the $\rho\rho$ model $w(1) = \frac{2}{3}$ and $w(2) = \frac{1}{3}$.

(ii) The cluster decay is a truncated Poisson distribution, i.e., $w(0) = 0$. Then $\lambda(z) = (1 - e^{-p_0})^{-1} (e^{zp_0} - 1)$ and one finds immediately from relations (17) and (18) that:

$$P_1(n) = p_0 (n-1)\, P(n-1)\,,$$

and consequently:

$$r(n) = \frac{\langle p(p-1) \rangle_n}{\langle p \rangle_n} = p_0 \frac{n-1}{n} \frac{P(n-1)}{P(n)}\,. \tag{20}$$

This formula was first derived by Berger [16].

It is thus clear that the experimental determination of the ratio $\langle p(p-1) \rangle_n / \langle p \rangle_n$ is an important tool in investigating cluster models. While its magnitude relates to the properties of the cluster decay distribution, its variation with n gives an important clue as to what extent clusters are produced independently. This variation is strongest when clusters are non-independent. This point is demonstrated in figs. 3a, b where in each case, fig. 3a for the $\rho\rho$ model, fig. 3b for the truncated Poisson model, we compare the $r(n)$ obtained on one hand in an independent cluster model, as done in sect. 3, and on the other hand, derived directly from the topological cross sections. To do the latter calculation, we assume a Poisson distribution for negative particles, in agreement with experimental data at 205 GeV on the non-diffractive component [14], and use the expressions given above in (i) and (ii).

The previous remarks can also be understood looking at f_2, the second Mueller moment. At 205 GeV, $f_2^- = 0$ for the non-diffractive component [14]. This implies that f_2^{cluster} is negative since by integration of the inclusive two-particle correlation we have

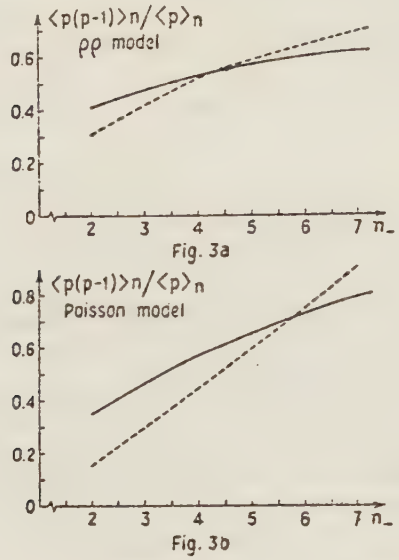

Fig. 3. The ratio $\langle p(p-1)\rangle/\langle p\rangle$ as a function of n. (a) The ρ-ρ model. (b) The Poisson model for cluster decay. The full lines correspond to independent cluster emission, the dashed lines to a Poisson distribution for negative particles.

$$f_2 = f_2^{\text{cluster}} \langle p\rangle^2 + \bar{k}\,\langle p(p-1)\rangle \ .$$

We have checked that a negative value for f_2^{cluster} leads to an increase with n of $r(n)$ which is faster than in the case of independent clusters, except of course, when the number of charged particles in a cluster is fixed. Therefore, in general, in cluster models R_n is expected to fall off with n more slowly than $1/n$.

6. Summary

We here summarize the main points made in the preceeding sections. We first established a general formula (eq. (4)), valid in all short-range order models, that expresses the semi-inclusive correlation function in terms of the inclusive one. We then gave the specific form (eq. (7)) this formula takes for $R_n(0, 0)$ in the case of neutral cluster models. The expression for $R_n(0, 0)$ depends essentially on the ratio $\langle p(p-1)\rangle_z/\langle p\rangle_z$, which is characteristic of cluster properties. A sensitive test of cluster models therefore consists in extracting from the data this quantity. However, though eq. (7) is true in all cluster models, it is useful only when clusters are produced independently, in which case the relation between the variable z and the number n of particles is simple. When clusters are not independent, it is advantageous to relate $\langle p(p-1)\rangle_n/\langle p\rangle_n$, directly to the topological cross sections. Thus, in all cases, the determination of the ratio $\langle p(p-1)\rangle_n/\langle p\rangle_n$, in particular its variation with n, is crucial if one wants to learn about clusters, their decay mechanism and their production.

We wish to thank Ed. Berger for stimulating discussions.

References

[1] E.L. Berger, CERN preprint (1974) TH 1816.
[2] J. Ranft and G. Ranft, Phys. Letters 49B (1974) 286.
[3] A. Biatas, Invited talk at the 4th Conf. on hadrodynamics, Pavia, CERN preprint (1973) TH 1745.
[4] R.C. Arnold and G.H. Thomas, ANL preprint (1974) HEP 7401.
[5] L. Caneschi, CERN preprint (1974) TH 1826.
[6] R. Singer et al., ANL preprint (1974) HEP 7369.
[7] G. Bellettini, Invited talk at the Stony Brook Conf. AIP Conf. Proc., ed. C. Quigg (1973).
[8] W. Frazer, R. Peccei, S. Pinsky and C.I. Tan, Phys. Rev. D7 (1973) 2647.
[9] M. Le Bellac, Acta Phys. Pol. B4 (1973) 901.
[10] P. Pirilā and S. Pokorski, Phys. Letters 43B (1973) 502;
F. Hayot and A. Morel, Saclay preprint (June 1973); Nucl. Phys. B68 (1974) 323.
[11] A. Biatas, K. Fialkowski and K. Zalewski, Phys. Letters 45B (1973) 337.
[12] L. Foá, Invited talk at the 9th Moriond meeting (1974).
[13] F. Hayot and A. Morel, Nuovo Cimento Letters 7 (1973) 221; 8 (1973) 71.
[14] R. Singer et al., ANL preprint HEP 7367 (1973).
[15] K. Jaeger et al., Results presented at the AIP Meeting on particles and fields, AIP Conf. Proc., ed. H. Bingham (1973).
[16] E.L. Berger, private communication.

REPRINT N°6

RESONANCES IN HIGH-MULTIPLICITY REACTIONS .

LETTERE AL NUOVO CIMENTO VOL. 12, N. 17 26 Aprile 1975

Resonances in High-Multiplicity Reactions (*).

F. HAYOT (**)

Department of Physics, University of Michigan - Ann Arbor, Mich.

(ricevuto il 12 Febbraio 1975)

Cluster models have proved to be quite useful in the description of high-multiplicity reactions (1). Those models in which clusters are produced independently appear quite naturally as a generalization of multiperipheral-type models with a Poisson distribution for the final-state particles. In particular, they describe quite well the short-range part of the two-particle rapidity correlations.

Whether clusters are in fact dynamically produced has remained an open question. From the study of inclusive two-particle correlations (2) and one-particle transverse distributions (3), it appeared that, if they exist, clusters may be rather heavy objects (mass of 1 to 1.5 GeV) and decay on the average into more than two charged particles. In particular, the so-called $\rho\rho$ model (4) (each cluster decays with a probability $\frac{2}{3}$ into $\pi^+\pi^-\pi^0\pi^0$ and a probability $\frac{1}{3}$ into $2\pi^+2\pi^-$) leads generally to a satisfactory description of the short-range features present in the data.

Recent developments, however, seem to suggest that the clusters are not new objects reflecting some unexplored dynamical mechanism, but are simply resonances, such as ρ's and ω's. In this way, many of the aspects of particle production at high energies, which consists mainly of pions, can be attributed to the fact that these pions originate from well-known resonances. Two recent results seem to point in the direction of such an interpretation:

i) The observation (5) of a ρ_0 signal in the invariant mass of π^+ and π^- in the reaction $\pi^-p \to \pi^+\pi^-X$ at 205 GeV, with the estimate that ρ^0 production accounts for about 13% of all π^-'s observed in the central region.

(*) Work supported by the U.S. Atomic Energy Commission.

(**) Permanent address: Service de Physique Théorique, C.E.N., Saclay.

(1) For a review see A. BIALAS: invited talk at the *IV Conference of Hadrodynamics (Pavia)*, CERN preprint (1973) TH-1745.

(2) P. PIRILA and S. POKORSKI: *Phys. Lett.*, 43 B, 502 (1973); F. HAYOT and A. MOREL: *Nucl. Phys.*, 68 B, 323 (1974).

(3) S. POKORSKI and L. VAN HOVE: CERN preprint TH 1772 (1973).

(4) F. HAYOT and A. MOREL: *Lett. Nuovo Cimento*, 7, 221 (1973); 8, 71 (1973).

(5) F. C. WINKELMANN: presented at the *IV International Conference on Experimental Meson Spectroscopy, Boston, 1974.*

ii) The magnitude and variation with prong number of the short-range two-particle rapidity correlation observed at the ISR. These are data at $\sqrt{s} = 63$ GeV from the Pisa-Stony Brook ([6]) group and at $\sqrt{s} = 53$ GeV from the ACHM collaboration ([6]).

This second point needs explaining. It has been shown ([7,8]) that in all short-range models, the semi-inclusive two-particle rapidity correlation ([9]) C_n can be related to the exclusive one with some approximation at finite energies. As a result, in cluster models specifically, the width of the correlation in C_n is predicted to be independent of n. This seems to be borne out by the data and therefore a description in terms of clusters is favored. The relevant quantity extracted from the data which characterizes the cluster decay into p particles is

$$r(n) = \frac{\langle p(p-1)\rangle_n}{\langle p\rangle_n} .$$

The ISR data seem to indicate that $r(n)$ is independent of n. This results ([8]) if each cluster decays into a fixed number of particles. In this case the above-mentioned relation between C_n and C becomes exact and the magnitude of $r(n)$ can be determined, without any ambiguity attached to the derivation of the relation itself. The data, if they will hold up, seem to indicate a value very close to 1. In summary then

$$r(n) = p - 1 \simeq 1 .$$

(The result of the ACHM collaboration quoted in ref. ([6])) is $r(n) = 1.2 \pm 0.1$.)

Thus, a picture emerges in which each cluster decays into exactly two charged particles. Since according to point i) the ρ^0 makes an important contribution to the central π^-'s, it is only natural to interpret point ii) in the same context, namely clusters are resonances.

The over-estimate of the cluster size from inclusive rapidity correlations C can be attributed to the fact that C is not only built up from all the C_n's, but also from a crossed term depending on the difference of inclusive and semi-inclusive single-particle distributions (cf. footnote ([9])).

If clusters are resonances, then one must ask which resonances dominate many-particle production. An important part of the answer to this question must have to do with the neutral particles. One knows from ISR data that inclusive rapidity correlations between a charged and a neutral particle are as large as between two charged ones. More information relevant to this point should become available as soon as data on neutral charged rapidity correlations at *fixed* prong number will become available. These, in the same way as for $r(n)$, will lead to the determination of the quantity ([8])

$$r_0(n) = \frac{\langle p_0 p\rangle_n}{\langle p\rangle_n} .$$

([5]) Quoted by W. T. Ko: review presented at the *XVII International Conference on High-Energy Physics, London, July 1974*, University of California, Davis, preprint UCD-PPL-7-17-74.

([7]) R. C. Arnold and G. H. Thomas: ANL preprint (1974) HEP 7401.

([8]) F. Hayot and M. LeBellac: University of Michigan preprint (1974), UM HE-74-7, to be published in *Nucl. Phys.*, B; A. Morel and G. Plaut: *Nucl. Phys.*, 78 B, 541 (1974).

([9]) The extraction of the short-range component from C_n is made difficult because this correlation which is parametrized in the central region as a Gaussian in $y_1 - y_2$ of whith 1 sits upon a much larger background with Gaussian shape of width of about 2 centered around $y = 0$.

Neutral and charged particles are correlated, either because they originate from the same resonances, and/or because the resonances themselves they come from are correlated. In the former case ω production could be important. However, ω production by itself cannot explain all of the present data, at least not if resonances are produced independently. The reason is that the measured quantity $f_2^{00} = \overline{n_0(n_0-1)} - \overline{n}_0^2$ is large and therefore π^0's are correlated among themselves. In the second case, however, this effect can be easily explained in a model where ρ^+ and ρ^- production occurs (besides ρ^0 production) and the two charged ρ's are correlated through local charge conservation. That charge is conserved locally over an average rapidity distance of about 0.8 has been checked experimentally at $p_{\text{lab}} = 69$ GeV/c ([10]), and is consistent with the data on charge transfer fluctuations ([8]). Using the language of cluster models, one then obtains a description in which clusters are *not* neutral and are no longer produced independently because of local charge conservation.

Nevertheless, taking into account the fact that ρ^+ and ρ^- are produced nearby in rapidity, one can attempt to describe those correlations obtained by integrating rapidities in a model in which the ρ^+ and ρ^- are linked together and treated as a single object. This leads to a model with independent production where half of the time the object consists of $\rho^+\rho^-(=\pi^+\pi^-\pi^0\pi^0)$ and the other half of $\rho^0(=\pi^+\pi^-)$. Such a model is very similar to a $\omega(=\pi^+\pi^-\pi^0)$ model, except that it leads to correlation between $2\pi^0$'s ([11]). Both models lead to the same value equal to 1 for $r(n)$ *and* $r_0(n)$, independent of n. (In the so-called $\rho\rho$ model, an increase of $r(n)$ of about 20% is predicted ([8]) when n increases from $\frac{2}{3}\bar{n}$ to $1.5\,\bar{n}$ and, most importantly, $r(n=\bar{n})=2$.)

The preceding exercise indicates that, in practice, it will of course be very difficult to differentiate between the decay mode $\pi^+\pi^-2\pi^0(=\rho^+\rho^-)$ of a presumed neutral cluster and a pair of ρ^+ and ρ^- resonances correlated through local charge conservation. In principle, however, there is an important dynamical distinction. In the latter case the dynamical mechanism for correlation is clearly spelled out, while in the former case it is not, the dynamics of the formation of the so-called cluster remaining obscure. It is doubtful whether much new could be learned from the increasing amount of data, if their description a language, that of clusters, is used which takes into account only very crudely the actual dynamical mechanism. This mechanism appears to be related to the presence of low-lying resonances correlated through local charge conservation.

* * *

I thank R. THUN and C. QUIGG for discussions.

([10]) J. DERRE *et al.*: submitted to the *XVII International Conference on High-Energy Physics, London, July 1974*.

([11]) One finds in particular $f_2^{00} = \overline{n_0(n_0-1)} - \overline{n}_0^2 = \overline{n}_-$. Experimentally ([4]) at 300 GeV/c $f_2^{00} = 1.8 \pm 0.6$, $\overline{n}_- \simeq 3.5$.

REPRINT N°7

CLUSTERS, CORRELATIONS AND TRANSVERSE MOMENTA

REPRINT N°7

Nuclear Physics B80 (1974) 77–89. North-Holland Publishing Company

CLUSTERS, CORRELATIONS AND TRANSVERSE MOMENTA

F. HAYOT *, F.S. HENYEY and M. LEBELLAC **
Department of Physics, University of Michigan, Ann Arbor, Michigan 48104 ***

Received 4 June 1974

Abstract: We discuss the short range part of two-particle correlations as it results from the phase space available in cluster decay. In such an approach, certain variables emerge which should be useful to organize the data and extract interesting information. We are in particular concerned with tests of large transverse motion of clusters.

1. Introduction

The observed short range rapidity correlations between two particles have been often interpreted in terms of clustering [1–5]. Aside from a rough comparison with the shape of the correlation, this interpretation has been without a test. The magnitude of the correlation is fit by adjusting the cluster size.

Recently the idea of clustering has also been used to describe the single particle inclusive transverse momentum distribution. Again the distribution of cluster masses is adjusted to fit the transverse momentum dependence of the inclusive cross section [6]. In fact there is so much freedom that the additional assumption of no transverse cluster motion can be made.

Thus one has to study more detailed data if one wishes to test the clustering hypothesis. The ultimate goal of course is to be able to tell whether clustering is merely a convenient language, or whether clusters are actually produced. It therefore becomes of interest to determine the cluster parameters further and use for this purpose the full dependence of the two-particle correlation on the four kinematical variables, and not only its dependence on rapidity differences.

One of the most important points we wish to address ourselves to in this paper is the question of whether clusters can have a large transverse momentum. In fact the idea of clustering has been used to circumvent the difficulties of two standard models of multiparticle production with respect to elastic scattering [7–9]. It is known that the independent emission model (IEM) and the multiperipheral model

* Permanent address: Service de Physique Théorique, C.E.N. Saclay.
** Permanent address: Faculté des Sciences, Université de Nice.
*** Work supported by the US Atomic Energy Commission.

(MPM) give wrong predictions for the overlap function, if these models are used to describe the production of particles. This point will be discussed in more detail in sect. 2.

In sect. 3 we study two-particle correlations as they result from cluster phase-space. We point out some characteristic features, related both to a reasonable choice of variables with which to organize the data, and ways to study cluster transverse motion.

The general framework which we adopt is the following: we assume that in a typical high energy collision several clusters of particles are produced via an unspecified dynamical mechanism. We assume the existence of a central region in rapidity, where only rapidity differences between final particles matter (short range order). In this region, clusters are produced uniformly in rapidity and their production can be specified as a function of the cluster mass M and transverse momentum $K_\perp$. The clusters decay independently of production, except that, of course, the decay depends on the cluster mass. In particular, we assume that clusters decay isotropically in their rest frame.

As sect. 3 is unfortunately rather technical, we finally summarize in sect. 4 those aspects and results of our work which can be of interest to experimentalists.

2. Elastic scattering and cluster transverse momentum

In a cluster model, the transverse momentum $p_\perp$ of an observed particle is the combination of the cluster transverse momentum $K_\perp$ and of the particle transverse momentum $q_\perp$ in the cluster rest frame. In order to get a feeling about the order of magnitude involved, we first derive a general formula which relates $\langle p_\perp^2 \rangle$ to $\langle K_\perp^2 \rangle$ and $\langle q_\perp^2 \rangle$. This formula will allow us to define what is a typically small value of $\langle K_\perp^2 \rangle$ and what is on the contrary a typically large value of $\langle K_\perp^2 \rangle$. Interestingly enough, these two cases will be associated with the independent emission model and the multiperipheral model respectively.

Consider a cluster that is boosted from rest to a momentum $K_\perp$ along the x-direction (the z-axis being, as usual, the axis defined by the incident particles). If η is the cluster transverse rapidity and $M_\perp = \sqrt{M^2 + K_\perp^2}$ the cluster transverse mass:

$$M_\perp = M \cosh \eta, \quad K_\perp = M \sinh \eta, \tag{1}$$

the particle momentum is given after the boost by:

$$p_x = q_x \cosh \eta + E \sinh \eta,$$

$$p_y = q_y.$$

Isotropy of the cluster decay gives

$$\langle q_x \rangle = 0, \quad \langle q_x^2 \rangle = \langle q_y^2 \rangle = \tfrac{1}{2} \langle q_\perp^2 \rangle = \tfrac{1}{3} \langle E^2 - \mu^2 \rangle.$$

Averaging first over the decay distribution, and then over cluster production (which is possible since both processes are independent) we get:

$$\langle p_\perp^2 \rangle = \langle q_\perp^2 \rangle (1 + 2 \langle K_\perp^2/M^2 \rangle) + \mu^2 \langle K_\perp^2/M^2 \rangle, \tag{2}$$

where μ is the pion mass. The last term in (2) can be usually neglected; it would become of importance only if the particles were almost at rest in the cluster rest frame, which would have been easily discovered experimentally.

As a rough estimate of (2), we can choose *

$$\langle q_\perp^2 \rangle \sim \langle E \rangle^2 \sim \frac{\langle M \rangle^2}{\langle n \rangle^2},$$

where $\langle n \rangle$ is the average number of particles within a cluster. Experimentally $\langle p_\perp^2 \rangle \sim 0.2\ (\mathrm{GeV})^2$ while various estimates indicate that $\langle n \rangle \sim 4$ is a fair guess [2, 3, 5]. We then obtain in the two cases, $\langle K_\perp^2 \rangle = 0$ and $\langle K_\perp^2 \rangle = 1\ \mathrm{GeV}^2$:

(i) $\langle K_\perp^2 \rangle = 0$, $\langle q_\perp^2 \rangle = 0.2\ \mathrm{GeV}^2$, $M \simeq 1.8\ \mathrm{GeV}$,

(ii) $\langle K_\perp^2 \rangle = 1\ \mathrm{GeV}^2$, $\langle q_\perp^2 \rangle = 0.075\ \mathrm{GeV}^2$, $M \simeq 1.1\ \mathrm{GeV}$.

It is clear from (2) that a cluster transverse momentum is large whenever it is of the same order as the cluster mass. We wish now to show that cases (i) and (ii) can be associated with IEM and MPM respectively.

In IEM, assuming that the leading clusters (or particles) carry off most of the available angular momentum, it is easy to show, either by a direct calculation, or from the impact parameter analysis of ref. [7], that the overlap function is proportional to

$$\exp\left(\frac{t}{2 \langle K_\perp^2 \rangle}\right). \tag{3}$$

From data on elastic scattering, we know that the argument in the exponential is $\sim 6t$, so that

$$\langle K_\perp^2 \rangle \sim \tfrac{1}{12} \sim 0.08\ \mathrm{GeV}^2,$$

which must be considered as a small number **: indeed we can see from (2) that with such a value of $\langle K_\perp^2 \rangle$, and with $M^2 \sim 3$ most of the particle transverse motion is accounted for by the motion within the cluster.

On the other hand, MPM correspond to a random walk in impact parameter space. In simple versions this leads to a proportionality, as a function of the incident energy, between the elastic slope and the number of clusters minus one, which is in

* A more accurate calculation will be presented in sect. 3, where we use a specific model of cluster decay.

** As is well-known, inclusion of phases would lead to a larger value of $\langle K_\perp^2 \rangle$, and our estimate is a lower bound.

contradiction with the data. In order to remedy this, Caneschi [10] has suggested that the first and last step in the random walk are several times as large as the central steps. This adds a constant A_1 to the elastic slope B which is given by:

$$B \simeq A_1 + \langle N\rangle/(\langle n\rangle \langle K_\perp^2\rangle), \tag{4}$$

where $\langle N\rangle$ is the average number of particles produced in the collision. Since the multiplicity and the slope are logarithmic in MPM, we can identify the coefficients of $\log s$ in (4) with that in the usual expression

$$B = A_1' + 2\,\alpha_p' \log s\,, \tag{5}$$

where $\alpha_p' \simeq 0.25\ (\mathrm{GeV})^{-2}$. The coefficient of $\log s$ in $\langle N\rangle$ is of the order of 2.5, so that we get by comparison of (4) and (5):

$$\langle K_\perp^2\rangle \sim \frac{5}{\langle n\rangle} \sim 1.2\ \mathrm{GeV}^2\,.$$

This value is again a lower estimate, but we see that in any case MPM is to be associated with typically large values of $\langle K_\perp^2\rangle$.

To summarize: We can say that the cluster transverse momentum is small (large) whenever $2(K^2/M^2)$ in (2) is smaller (larger) than one. Provided our order of magnitude arguments are not spoiled by subleties about phases etc. ..., we can associate small $\langle K_\perp^2\rangle$ with IEM, large $\langle K_\perp^2\rangle$ with MPM.

3. One- and two-particle distributions in cluster models

Let us state again what we exactly mean by cluster models. Our first assumption is that clusters are produced independently. The implication of azimuthal or long range correlations between clusters will be treated later. The short range part of the correlation between two particles is then obtained when both particles originate from the same cluster.

We next assume that phase-space controls the properties of cluster decay (more generally we could assume a smoothly varying matrix element leading to isotropic decay). If the description in terms of cluster production and decay is not merely a convenient language, but corresponds to a true physical mechanism, our assumption means that the dynamics of the multiparticle process is buried in the cluster formation, whereas the two-particle correlation is determined from the kinematics of the decay alone.

3.1. General discussion

Let us concentrate on one cluster. The probability that a cluster with mass M decays into n particles is proportional to

$$\Gamma_n(K^2) = \int \frac{\mathrm{d}^3 p_1}{E_1} \cdots \frac{\mathrm{d}^3 p_n}{E_n}\, \delta^{(4)}\left(K - \sum_1^n p_i\right), \tag{6}$$

where K is the cluster four-momentum ($K^2 = M^2$). The Lorentz invariant one-particle distribution F is then given by:

$$F((K-p)^2) = \frac{\Sigma_n n g_n \Gamma_{n-1}((K-p)^2)}{\Sigma_n g_n \Gamma_n(K^2)}, \tag{7}$$

where g_n is some coupling constant. A common choice is [6]

$$g_n = \frac{g^n}{n!},$$

but we do not want to make this choice here. We remark, for further use, that F is an increasing function of its argument. Similarly the two-particle distribution $\tilde{F}$ is given by:

$$\tilde{F}((K-p_1-p_2)^2) = \frac{\sum_n g_n n(n-1)\Gamma_{n-2}((K-p_1-p_2)^2)}{\sum_n g_n \Gamma_n(K^2)}, \tag{8}$$

Integration of F with the measure d^3p/E and of $\tilde{F}$ with the measure $d^3p_1 d^3p_2/E_1E_2$ give $\langle n\rangle$ and $\langle n(n-1)\rangle$ respectively. The main point about (7) and (8) is that the one and two particle distributions depend only on the missing mass $m^2 = (K-p)^2$ or $(K-p_1-p_2)^2$.

The cross section $\rho(p)$ for observing one particle of momentum p for the c.m. frame of the reaction is obtained by integrating (7) over the cluster mass, rapidity and transverse momentum distribution. Let $f(K_\perp, M)$ be the cross section for producing a cluster of mass M and transverse momentum $K_\perp$ normalized as follows:

$$\int dY\, d^2K_\perp\, dM\, f(K_\perp, M) = \langle n_{cl}\rangle, \tag{9}$$

where $\langle n_{cl}\rangle$ is the average number of clusters. Since we are interested in the central region, f is independent of the cluster rapidity Y. The missing mass $m^2 = (K-p)^2$ can be written as:

$$m^2 = M^2 + \mu^2 - 2M_\perp \mu_\perp \cosh Y + 2K_\perp p_\perp \cos\Phi, \tag{10}$$

where $\mu_\perp$ is the pion transverse mass, Φ is the azimuthal angle between $K_\perp$ and $p_\perp$, and Y is the difference between the pion and cluster rapidities. The one-particle cross section $\rho(p)$ is then given by:

$$\rho(p) = \int dM\, dY\, dK_\perp\, d\Phi\, K_\perp f(K_\perp, M) F(M^2 + \mu^2 - 2M_\perp\mu_\perp \cosh Y + 2K_\perp p_\perp \cos\Phi). \tag{11}$$

In order to write the two-particle distribution $\tilde{C}(p_1, p_2)$ we introduce the invariant mass $\tilde{\mu}$ and the transverse mass $\tilde{\mu}_\perp$ of the system formed by the two particles:

$$\tilde{\mu}^2 = (p_1 + p_2)^2, \tag{12a}$$

$$\tilde{\mu}_\perp^2 = \tilde{\mu}^2 + (p_{1\perp} + p_{2\perp})^2 = \tilde{\mu}^2 + \tilde{p}_\perp^2, \tag{12b}$$

the missing mass $m^2 = (K - p_1 - p_2)^2$ is then expressed as:

$$m^2 = M^2 + \tilde{\mu}^2 - 2M_\perp \tilde{\mu} \cosh Y + 2K_\perp \tilde{p}_\perp \cos \Phi. \tag{13}$$

It is then clear that the expression for $\tilde{C}(p_1, p_2)$ is identical to (11) with the substitutions $\mu \to \tilde{\mu}$, $\mu_\perp \to \tilde{\mu}_\perp$, $F \to \tilde{F}$. We recall that $\tilde{C}(p_1, p_2)$ is that part of the two-particle inclusive cross section where both particles originate from the same cluster. If the clusters are produced independently, $\tilde{C}(p_1, p_2)$ is the usual correlation. We now notice that $\tilde{C}(p_1, p_2)$ depends only on the two variables $\tilde{\mu}$ and $\tilde{\mu}_\perp$. Thus we can make the following important point: although the correlation in the central region depends a priori on four variables (for example $(y_1 - y_2)$, $p_{1\perp}$, $p_{2\perp}$, and the azimuthal angle φ), in a model of independent clusters decaying via phase space this correlation depends only on the two combinations $\tilde{\mu}$ and $\tilde{\mu}_\perp$ formed with these variables:

$$\tilde{\mu}^2 = 2\mu^2 + 2\mu_{1\perp}\mu_{2\perp} \cosh (y_1 - y_2) - 2p_{1\perp}p_{2\perp} \cos \varphi, \tag{14a}$$

$$\tilde{\mu}_\perp^2 = \mu_{1\perp}^2 + \mu_{2\perp}^2 + 2\mu_{1\perp}\mu_{2\perp} \cosh (y_1 - y_2). \tag{14b}$$

Since only μ^2 depends on the azimuthal angle φ, it is very easy to understand qualitatively the φ dependence of the correlation. Let us calculate the derivative with respect to $\tilde{\mu}^2$ of the correlation written as a function of $\tilde{\mu}^2$ and $\tilde{\mu}_\perp^2$:

$$\frac{\partial}{\partial \tilde{\mu}^2} \tilde{C}(\tilde{\mu}^2, \tilde{\mu}_\perp^2) = \int \mathrm{d}Y \, \mathrm{d}M \, \mathrm{d}P_\perp \, \mathrm{d}\Phi \, K_\perp \, f(K_\perp, M) \tag{15}$$

$$\times \left(1 - \frac{K_\perp}{\tilde{p}_\perp} \cos \Phi\right) \tilde{F}' (M^2 + \tilde{\mu}^2 - 2M_\perp \tilde{\mu}_\perp \cosh Y + 2K_\perp \tilde{p}_\perp \cos \Phi).$$

First it is obvious from (15) that if the clusters have negligible transverse motion, $\tilde{C}$ is an increasing function of $\tilde{\mu}^2$ since $\tilde{F}' \geqslant 0$: the two-particle correlation will then be larger at $\varphi = 180°$ than at $\varphi = 0°$. On the contrary, when the cluster transverse motion becomes large, $\tilde{C}$ will finally become a decreasing function of $\tilde{\mu}^2$ (since $\tilde{F}'$ is an increasing function the average value of $\cos \Phi$ is positive). The correlation will then be larger at 0° than at 180° *.

The physical interpretation is very clear: when the clusters are longitudinal, energy-momentum conservation in the cluster decay tends to favor particles going into opposite directions. However when the cluster transverse motion is large, both particles tend to follow its motion, and this effect finally wins over the first one.

3.2. A specific model for cluster decay

It seems difficult to go further without making some specific assumptions about cluster production and decay. Although these assumptions may turn out not to be

* A Monte-Carlo calculation by Berger and Ellis give the same result. We thank E. Berger for showing us the φ-dependence of the correlation in his calculation.

entirely correct, we believe that they provide a useful illustration of the general formalism and may give hints on ways to organize the experimental data.

Our first assumption concerns cluster decay. We assume that the function $F(m^2; M^2)$ is an exponential in m^2

$$F(m^2; M^2) \propto \exp\left(\frac{\lambda}{2M} m^2\right), \tag{16}$$

in order to obtain an exponential form for the particle distribution in the cluster rest frame:

$$F = \frac{\gamma}{4\pi} \exp(-\lambda E). \tag{17}$$

Since $m^2 = M^2 + \mu^2 - 2ME$. The normalization constant γ in (17) is calculated in appendix A. The behavior (16) or (17) can be obtained approximately by a suitable choice of the coefficients g_n in (7): for example in the approximation that the pions are massless we have [11]:

$$\Gamma_n(K^2) = \frac{2\pi^{n-1}}{(n-1)!\,(n-2)!} (K^2)^{n-2}, \tag{18}$$

and choosing $g_n = (\lambda/2\pi M)^{n-2}(n-2)!/n$ gives the behavior (16). For the two-particle distribution $\tilde{F}$ we also choose an exponential in m^2. Although this choice does not satisfy exactly the energy momentum sum rules, we believe that it displays the essential features that we need *.

With the choice (17) we can perform the Φ and Y integrations in (11), provided we forget about the limits of integration due to the positivity of m^2. Since the decay is exponential, this is an excellent approximation except for large values of $p_\perp$. We then get

$$\rho(p) = \gamma \int dM\, dK_\perp\, K_\perp f(K_\perp, M) K_0\left(\frac{\lambda \mu_\perp M_\perp}{M}\right) I_0\left(\frac{\lambda p_\perp K_\perp}{M}\right), \tag{19}$$

where I_0 and K_0 are modified Bessel functions. The calculation of the two-particle correlation is identical and leads to the result:

$$\tilde{C}(p_1, p_2) = \gamma \int dM\, dK_\perp K_\perp f(K_\perp, M)\, e^{\lambda \tilde{\mu}^2/2M} K_0\left(\frac{\lambda \mu_\perp M_\perp}{M}\right) I_0\left(\frac{\lambda \tilde{p}_\perp K_\perp}{M}\right). \tag{20}$$

Since $\tilde{p}_\perp = (\tilde{\mu}_\perp^2 - \tilde{\mu}^2)^{\frac{1}{2}}$, we see again from (20) that $\tilde{C}$ depends only on the variables $\tilde{\mu}$ and $\tilde{\mu}_\perp$. The term $\exp(\lambda\tilde{\mu}^2/2M)$ expresses the way in which the two particles are correlated in the cluster rest frame.

We can make two remarks about (20):

(i) In the case of longitudinal clusters, we may expect an approximate factoriza-

* In the case of massless pions, the choice $F(m^2) \propto (6 + 6\lambda' m^2 + \lambda'^2 m^4) \exp(\lambda' m^2)$ with $\lambda' = \lambda/2M$ satisfies exactly the energy momentum constraints.

tion of $\widetilde{C}$ as a function of $\widetilde{\mu}$ and $\widetilde{\mu}_\perp$, since I_0 disappears from the integrand in (20).

(ii) In the general case we can take the derivative of $\widetilde{C}$ with respect to $\widetilde{\mu}^2$, as in (15). The integrand is then proportional to

$$\left(1 - \frac{K_\perp I_1}{\widetilde{p}_\perp I_0}\right).$$

Unless $K_\perp$ is very small, the ratio I_1/I_0 will be of the order of unity, and we expect that $\widetilde{C}$ becomes a decreasing function of $\widetilde{\mu}^2$ as soon as $\langle K_\perp^2 \rangle$ is significantly larger than $\langle p_\perp^2 \rangle$.

To proceed further, we choose a reasonable form of the function $f(K_\perp, M)$ in order to be able to perform the integrations analytically. Despite the arbitrariness of our choice, we believe that some aspects of the final results will show general validity. With this hope in mind we do the following. We write $f(K_\perp, M)$ in the form

$$f(K_\perp, M) = \frac{h(M)}{M^2} g\left(\frac{K_\perp}{M}\right),$$

and choose for g the following expression:

$$g(x) = \delta \frac{e^{-A\sqrt{1+x^2}}}{(1+x^2)^{\frac{1}{4}}}, \tag{21}$$

with $x = K_\perp/M$. The normalization constant δ is given in appendix A. With the choice (21) for the cluster distribution the average cluster transverse momentum squared is

$$\langle K_\perp^2 \rangle = \langle M^2 \rangle \left(\frac{\Gamma(\frac{7}{2}, A)}{A^2 \Gamma(\frac{3}{2}, A)} - 1\right), \tag{22}$$

where $\Gamma(a, b)$ is the incomplete gamma function. We remark that the argument of the K_0 function in (19) is always larger than one (since $\lambda \gtrsim 6$) and we can replace K_0 by its asymptotic approximation with an error of a few percent. Then, if there is no unfortunate misprint in eq. (8. 6. 22) of Erdelyi et al. [12] we have:

$$\rho(p) = \sqrt{\frac{\pi}{2}} \frac{\gamma\delta}{\sqrt{\lambda n_\perp}} \frac{e^{-\sqrt{A^2+\lambda^2\mu^2+2A\lambda\mu_\perp}}}{\sqrt{A^2+\lambda^2\mu^2+2A\lambda\mu_\perp}}. \tag{23}$$

The two-particle correlation is given by a formula analogous to (23), multiplied by $\exp(\lambda\widetilde{\mu}^2/2M)$.

In order to show that (23) gives a reasonable one-particle distribution we have drawn $\rho(p)$ on fig. 1 for two typical values of the parameters:

(i) typically small value of $\langle K_\perp^2 \rangle$: we choose $A = 25$, $\lambda = 6(\text{GeV})^{-1}$ so that

$$\langle K_\perp^2 \rangle = 0.085 \langle M^2 \rangle.$$

If we assume a sharp M distribution, the energy sum rule fixes $\langle M \rangle$ and $\langle M^2 \rangle$: we find $\langle M^2 \rangle \sim 3$ GeV^2 and

$$\langle K_\perp^2 \rangle \simeq 0.25 \text{ GeV}^2.$$

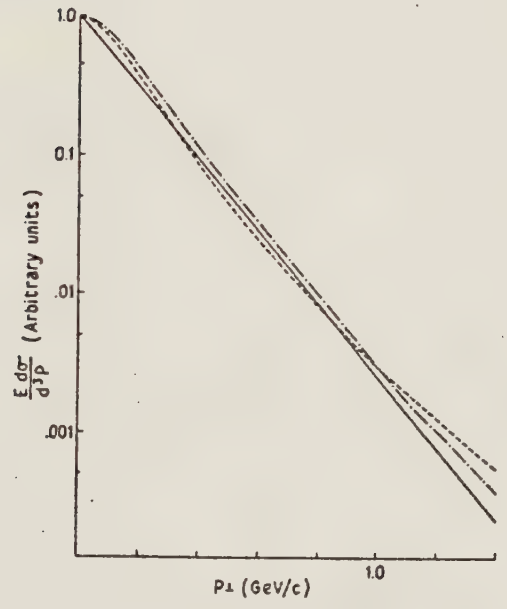

Fig. 1. $E\,d\sigma/d^3p$ as a function of $p_\perp$. Dashed line: $\langle K^2\rangle = 1.2$ GeV^2 [$A = 4$, $\lambda = 9$ in eq. (21)]. Dotted, dashed line: $\langle K_\perp^2\rangle = 0.25$ GeV^2 [$A = 25$, $\lambda = 6$]. The curve exp $(-6p_\perp)$ is given for comparison (full line).

(ii) typically large value of $\langle K_\perp^2\rangle$: we choose $A = 4, \lambda = 9$ GeV^{-1} so that:

$$\langle K_\perp^2\rangle = 0.70\,\langle M^2\rangle \simeq 1.2 \text{ GeV}^2 ,$$

since we have $\langle M^2\rangle \sim 1.7$ GeV^2. In this latter case it is interesting to notice the deviation of $\rho(p)$ from an exponential when $p_\perp \sim 1$ GeV. This deviation results from a completely different mechanism from that of Pokorski and Van Hove [6] who attribute it to large mass clusters. Here we obtain it as a consequence of the large transverse momentum of the clusters.

It is interesting to compute the correlation at $y_1 = y_2 = 0$ in both cases as a function of the azimuthal angle φ. If we fix $p_{1\perp}$ and $p_{2\perp}$ at their average value we immediately obtain from (23) the asymmetry:

$$A_s = \frac{\tilde{C}(\varphi = 0^\circ) - \tilde{C}(\varphi = 180^\circ)}{\tilde{C}(\varphi = 0^\circ) + \tilde{C}(\varphi = 180^\circ)} ;$$

we find

$$A_s = -0.25$$

in the first case (small $\langle K_\perp^2\rangle$) and

$$A_s = +0.29$$

in the second case (large $\langle K_\perp^2 \rangle$) in agreement with the qualitative discussion given previously.

3.3. Remarks on cluster correlations

We conclude this section by some comments on the complications which arise when the clusters are not independent. Then the correlation defined as usual by

$$C(p_1, p_2) = \frac{1}{\sigma_{in}} \frac{E_1 E_2 d\sigma}{d^3 p_1 d^3 p_2} - \frac{1}{\sigma_{in}} \left(\frac{E_1 d\sigma}{d^3 p_1} \right) \frac{1}{\sigma_{in}} \left(\frac{E_2 d\sigma}{d^3 p_2} \right) \tag{24}$$

is not equal to $\tilde{C}(p_1, p_2)$, since particles coming from different clusters are now correlated. There are (at least) two reasons why we expect some correlations between clusters: (i) existence of long range rapidity correlations due, for example, to a two-component mechanism for cluster production and (ii) existence of azimuthal correlations between clusters because of transverse momentum conservation.

Both effects are difficult to estimate, because they turn out to be strongly model dependent. Let us first examine the influence of long range rapidity correlations. A first step in order to minimize their influence would be to use as normalization in (24) the non-diffractive cross section σ_{ND} instead of the total inelastic cross section σ_{in}, or to use a specific model for diffraction [3, 5] in order to calcualte explicitly the long range part of the correlations. Either procedure probably allows to determine the short range component within a 10–15% error.

The problem of azimuthal correlations between clusters is serious only if the clusters have a large $\langle K_\perp^2 \rangle$. Assuming an IEM of cluster production we have a two-cluster inclusive cross section of the form [13]:

$$\frac{1}{\sigma} \frac{K_{10} K_{20} d\sigma}{d^3 K_1 d^3 K_2} \cong f(K_{1\perp}) f(K_{2\perp}) (1 - \alpha K_{1\perp} K_{2\perp} \cos\theta), \tag{25}$$

where θ is the azimuthal angle between the clusters and $\alpha \simeq 2/(\langle n_{cl} \rangle \langle K_\perp^2 \rangle)$. If the production mechanism is of the MPM type, α will be a decreasing function of the rapidity difference between the clusters, which means that the azimuthal correlation at zero rapidity difference is larger than in (25), since the transverse momentum sum rule [14] must be satisfied in either case. Hence we see that azimuthal correlations between clusters are model-dependent. In order to get an order of magnitude of the effect, we nevertheless assume (25), since we have no other reliable formula in the MPM case.

It is shown in appendix B that the correlation takes the following form:

$$C(p_1, p_2) = \rho(p_{1\perp}) \rho(p_{2\perp}) \{ -\alpha p_{1\perp} p_{2\perp} \hat{\rho}(p_{1\perp}) \hat{\rho}(p_{2\perp}) \cos\varphi + \tilde{C}(p_1, p_2) / \rho(p_{1\perp}) \rho(p_{2\perp}) \}. \tag{26}$$

The coefficient of $\cos\varphi$ in (26) is typically of the order of 20% when $\langle K_\perp^2 \rangle$ is large and $p_{1\perp}, p_{2\perp}$ are close to their average values. We can therefore hope that two-

particle correlations are not significantly affected by azimuthal cluster correlations.

4. Summary and conclusions

We here summarize the two main points of our paper which we believe should be of interest to experimentalists:

(i) If the proper amount of the product of single-particle inclusive cross section has been removed from the two-particle cross section, the remaining correlation function should depend mostly on $\tilde{\mu}$ and $\tilde{\mu}_\perp$ defined in eq. (14). This is important because the two particle inclusive cross sections depend generally on four variables in the central region (for example, $\tilde{\mu}, \tilde{\mu}_\perp, p_{1\perp}, p_{2\perp}$) and not just on two. Therefore, to test whether the short range part of the two-particle correlation is mainly determined by the phase space of cluster decay, one should organize the data in terms of different variables in order to find out whether the variation of the two-particle correlation is for the most part dependent on the two variables $\tilde{\mu}$ and $\tilde{\mu}_\perp$ only.

(ii) If there is transverse motion of clusters, this would indeed be an important indication that cluster formation is an essential ingredient in high energy multiparticle reactions. If it turns out that the variation with $\tilde{\mu}$ and $\tilde{\mu}_\perp$ accounts for the major part of the short range two particle correlation then the following test for cluster transverse momentum can be performed: hold $\tilde{\mu}_\perp$ fixed and vary $\tilde{\mu}$. Then the correlation function is predicted to rise if $\langle K_\perp \rangle$ is small and to fail if $\langle K_\perp \rangle$ is large. Another test is provided by the dependence of the correlation function on the azimuthal angle φ at $y_1 = y_2 = 0$: for $\langle K_\perp \rangle$ small the correlation is larger at $\varphi = 180^\circ$ than at $\varphi = 0^\circ$, while for $\langle K_\perp \rangle$ large the opposite is expected.

Though this work is a rather crude attempt towards the understanding of transverse momentum correlations, we hope that it will provide the impetus for looking at the data in more imaginative ways.

Appendix A. Normalizations

In this appendix we give for completeness the normalization constants γ and δ which appear in the one-particle distribution (17) and in the cluster distribution (21).

If the one-particle distribution within a cluster is given by (17) we have for the normalization constant

$$\gamma = \frac{\langle n \rangle \lambda\mu}{\mu^2 K_1(\lambda\mu)}, \tag{A.1}$$

so that in the cluster rest frame:

$$\int \frac{E}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}^3 q} \frac{\mathrm{d}^3 q}{E} = \langle n \rangle .$$

The average particle energy $\langle E\rangle$ in the cluster rest frame is

$$\langle E\rangle = \frac{1}{\lambda}\left(2 + \lambda\mu K_0(\lambda\mu)/K_1(\lambda\mu)\right) \tag{A.2}$$

and from the energy sum rule we have $\langle M\rangle = \langle n\rangle\langle E\rangle$. The average transverse momentum squared $\langle q_\perp^2\rangle$ which is needed in (2) is given by

$$\langle q_\perp^2\rangle = \tfrac{2}{3}\langle E^2 - \mu^2\rangle = \frac{2}{\lambda}\langle E\rangle. \tag{A.3}$$

In order to compute the constant δ in (21) we evaluate the integral

$$\int f(K_\perp, M)\, \mathrm{d}^2K_\perp \mathrm{d}M\, \mathrm{d}Y = 2\pi\delta \int h(M)\, \mathrm{d}M\, \mathrm{d}Y \int x\mathrm{d}x\, \frac{\mathrm{e}^{-A\sqrt{1+x^2}}}{(1+x^2)^{\frac{1}{4}}}$$

$$= 2\pi\delta A^{-\frac{3}{2}}\Gamma(\tfrac{3}{2}, A)\int h(M)\, \mathrm{d}M\mathrm{d}Y, \tag{A.4}$$

where $\Gamma(a, b)$ is the incomplete gamma function. Thus:

$$\delta = A^{\frac{3}{2}}\langle n_{\mathrm{cl}}\rangle\, [2\pi\, \Gamma(\tfrac{3}{2}, A)\int h(M)\, \mathrm{d}M\, \mathrm{d}Y]^{-1}\,, \tag{A.5}$$

so that

$$\int f(K_\perp, M)\, \mathrm{d}^2K_\perp\, \mathrm{d}M\, \mathrm{d}Y = \langle n_{\mathrm{cl}}\rangle.$$

The average value $\langle M^2\rangle$ used in (22) is to be defined with respect to $h(M)$:

$$\langle M^2\rangle = \left(\int h(M)\, M^2\, \mathrm{d}M\right)\left(\int h(M)\, \mathrm{d}M\right)^{-1}.$$

Appendix B. Transverse correlations between clusters

We assume that the two-cluster inclusive cross section is given by (25) and that the one-particle distribution within a cluster is exponential (17). A straightforward calculation leads to the following result:

$$p_\perp\rho(p_\perp)\hat{\rho}(p_\perp) = \gamma\int \mathrm{d}K_\perp K_\perp^2 K_0\left(\frac{\lambda M_\perp \mu_\perp}{M}\right) I_1\left(\frac{\lambda K_\perp p_\perp}{M}\right), \tag{B.1}$$

where ρ is defined in eq. (26) [compare with (19)]. In order to simplify the notations, we have neglected the dependence upon the cluster mass. In order to estimate $\hat{\rho}$ we equate $K_\perp$ to its average value in (B.1) and (19), which should be valid if $p_\perp \sim \langle p_\perp\rangle$. We then find

$$p_\perp\hat{\rho}(p_\perp) = \frac{\langle K_\perp\rangle I_1(\lambda\langle K_\perp\rangle p_\perp/M)}{I_0(\lambda\langle K_\perp\rangle\, p_\perp/M)}. \tag{B.2}$$

For $\langle K_\perp\rangle \sim 1$ GeV we find that $p_\perp\hat{\rho}(p_\perp)$ is of the order of ~ 0.7 GeV, which means that the coefficient of $\cos\varphi$ in (26) is $\sim 20\%$ at ISR where $\langle n_{\mathrm{cl}}\rangle \sim 4$. We can make a

more complete calculation by using the form (21) of $f(K_\perp)$. We find

$$\hat{\rho}(p_\perp) = \frac{\lambda M}{(A + \lambda m_\perp)^2 + \lambda^2 p_\perp^2} \quad (1 + \sqrt{(A + \lambda m_\perp)^2 + \lambda^2 p_\perp^2}). \tag{B.3}$$

With the values $A = 4$, $\lambda = 6\ \mathrm{GeV}^{-1}$ and $p_\perp \sim 0.3$ GeV we find again that $p_\perp \hat{\rho}(p_\perp)$ is ~ 0.7, in agreement with the previous estimate.

References

[1] P. Pirila and S. Pokorski, Phys. Letters 43B (1973) 502.
[2] F. Hayot and A. Morel, Nucl. Phys. B68 (1974) 323.
[3] E. Berger and G. Fox, Phys. Letters 47B (1973) 162.
[4] W. Schmidt–Parzefall, Phys. Letters 46B (1973) 399.
[5] M. LeBellac, H. Miettinen and R.G. Roberts, Phys. Letters 48B (1974) 115.
[6] S. Pokorski and L. Van Hove, CERN preprint TH1772 (1973).
[7] F.S. Henyey, Phys. Letters 45B (1973) 363, 469.
[8] R. Hwa, Phys. Rev. D8 (1973) 1131;
C. Hamer and R. Peierls, Phys. Rev. D8 (1973) 1358.
[9] F.S. Henyey, University of Michigan preprint UM74–8.
[10] L. Caneschi, Private communication.
[11] F. Lurçat and P. Mazur, Nuovo Cimento 31 (1964) 140.
[12] A. Erdelyi et al., Tables of integral transforms, vol. II, (McGraw Hill, 1954).
[13] L. Caneschi, Proc. of the 7th Moriond Rencontre, ed. Tran Than Van, 1972.
[14] L.S. Brown, Phys. Rev. D5 (1972) 748.

REPRINT N°8

SUM RULES AND TRANSVERSE-MOMENTUM FLUCTUATIONS IN SEMI-INCLUSIVE REACTIONS

PHYSICAL REVIEW D VOLUME 12, NUMBER 9 1 NOVEMBER 1975

Sum rules and transverse-momentum fluctuations in semi-inclusive reactions*

F. Hayot[1]
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794
(Received 13 May 1975)

We derive sum rules and set up a formalism for transverse-momentum fluctuations in semi-inclusive reactions. We discuss the relevance of these fluctuations for the determination of the transverse-momentum structure of multiparticle events.

I. INTRODUCTION

There has recently been considerable interest in the notion of local compensation of quantum numbers in multiparticle states, as introduced by Krzywicki and Weingarten.[2] In particular, it has been shown[1] that local compensation of transverse momentum in rapidity space leads via the overlap function to a lower bound on the Pomeron slope. Models have been exhibited in which this bound is saturated.[2] Whether, for transverse momentum, local compensation actually occurs will be decided by the study of the transverse-momentum distribution of particles in multiparticle reactions. The main difficulty then lies in the fact that at high energy only semi-inclusive experiments, with the possible detection of one or several neutrals, can be performed, whereas in theory, in relationship to the overlap function, *all* final-state particles ought to be detected and their transverse momenta measured. The question therefore arises of what can be learned from the study of transverse momentum transfer in a semi-inclusive experiment, insofar as the general structure of the transverse-momentum distribution is concerned.[3] In the following we will only set up the framework in which such an answer will be forthcoming by giving a general formalism of transverse-momentum fluctuations in semi-inclusive reactions. Such an endeavour seems to be useful if one wants to overcome the present stage of crude estimates of the quantities involved in the overlap-function calculations and gain a broader perspective on the fundamental issue of whether actually transverse momentum is locally compensated in rapidity space. Moreover, the aim is to encourage the experimental study of transverse-momentum fluctuations and thus provide, at the least, additional constraints for models of two-particle distributions. It can of course be argued that since there now exist data on the rapidity and azimuthal dependences of two-particle correlation functions[4] the study of transverse-momentum fluctuations, where only part of this detailed information is used, has become superfluous. As an objection to this, we can only express our belief that in the present case too much detail has been confusing and that it is important to learn more about the general transverse structure of an n-prong event before detailed knowledge can be appreciated and detailed model calculations will be really useful. We will proceed from this point of view, and thus in Sec. II we will generalize to the semi-inclusive case the sum rules previously obtained for inclusive reactions.[5] This generalization is needed in order to derive the expressions for quantities related to transverse momenta. This is done in Sec. III, where such expressions are discussed and their relevance to experiments examined.

II. SUM RULES

In order to obtain sum rules for semi-inclusive reactions, we need the differential cross sections for producing n charged particles of momenta p_i and l neutral particles of momenta k_i:

$$\frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l}$$

which we normalize to

$$\int \frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l} dp_1 \cdots dp_n dk_1 \cdots dk_l = \sigma^{(n,l)} .$$

Here $\sigma^{(n,l)}$ denotes the total cross section for producing n charged and l neutral particles, and dp (or dk) is the invariant volume element which is equal to $dy\,d^2p_\perp$ in terms of rapidity y and transverse momentum $p_\perp$. The single-particle cross section for a charged particle in an n-prong event is then given by

$$\frac{d\sigma^{(n)}}{dp} = \sum_l n \int \frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l} dp_2 \cdots dp_n dk_1 \cdots dk_l , \tag{1}$$

where the factor of n results from the indistinguishability of the charged particles. Consider now the quantity

$$\int p_\mu \frac{d\sigma^{(n)}}{dp} dp = \sum_l \int \frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l} dp_1 \cdots dp_n dk_1 \cdots dk_l \sum_{i=1}^{n} p_i .$$

Using energy-momentum conservation,

$$\sum_{i=1}^{n} p_i = P - \sum_{i=1}^{l} k_i ,$$

where P is the total incoming momentum, we obtain the first sum rule:

$$\int p_\mu \frac{d\sigma^{(n)}}{dp} dp = P_\mu \sigma^{(n)} - \int k_\mu \frac{d\sigma^{(n)}}{dk} dk . \tag{2}$$

Here $d\sigma^{(n)}/dk$ is the cross section for observing one neutral particle of momentum k in an n-prong event. It is normalized to

$$\int \frac{d\sigma^{(n)}}{dk} dk = \bar{l}(n)\sigma^{(n)} , \tag{3}$$

where $\bar{l}(n)$ is the average number of neutrals for a fixed n. Starting now from the double-differential cross section for two charged particles of momenta p_1 and p_2,

$$\frac{d\sigma^{(n,l)}}{dp_1 dp_2} = n(n-1) \int \frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l} dp_3 \cdots dp_n dk_1 \cdots dk_l ,$$

we consider the quantity

$$\int p_{1\mu} \frac{d\sigma^{(n)}}{dp_1 dp_2} dp_1 = n \sum_l \int \frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l} dp_1 dp_3 \cdots dp_n dk_1 \cdots dk_l \sum_{i=1}^{n-1} p_{i\mu} . \tag{4}$$

Using again energy-momentum conservation,

$$\sum_{i=1}^{n-1} p_i = P - p_2 - \sum_{i=1}^{l} k_i ,$$

relation (4) can be written as

$$\int p_{1\mu} \frac{d\sigma^{(n)}}{dp_1 dp_2} dp_1 = (P - p_2)_\mu \frac{d\sigma^{(n)}}{dp_2} - n \sum_l l \int \frac{d\sigma^{(n,l)}}{dp_1 \cdots dp_n dk_1 \cdots dk_l} dp_1 dp_3 \cdots dp_n k_\mu dk\, dk_2 \cdots dk_l . \tag{5}$$

From (5) one derives straightforwardly the sum rule

$$\int p_{2\nu} p_{1\mu} \frac{d\sigma^{(n)}}{dp_1 dp_2} dp_1 dp_2 = P_\mu \int p_{2\nu} \frac{d\sigma^{(n)}}{dp_2} dp_2 - \int p_{2\mu} p_{2\nu} \frac{d\sigma^{(n)}}{dp_2} dp_2 - \int p_{2\nu} k_\mu \frac{d\sigma^{(n)}}{dp_2 dk} dp_2 dk . \tag{6}$$

Using (2) this sum rule takes the form

$$\int p_{2\nu} p_{1\mu} \frac{d\sigma^{(n)}}{dp_1 dp_2} dp_1 dp_2 = P_\mu P_\nu \sigma^{(n)} - \int p_{2\mu} p_{2\nu} \frac{d\sigma^{(n)}}{dp_2} dp_2 - P_\mu \int k_\nu \frac{d\sigma^{(n)}}{dk} dk - \int k_\mu p_{2\nu} \frac{d\sigma^{(n)}}{dp_2 dk} dp_2 dk . \tag{7}$$

What distinguishes sum rules (2) and (7) from the exclusive case[5] is the presence on their right-hand sides of cross sections involving the observation of a neutral particle in an n-prong event. The cross section $d\sigma^{(n)}/dp_2 dk$ appearing in (7) is normalized in the following way:

$$\int \frac{d\sigma^{(n)}}{dp_2 dk} dp_2 dk = n\bar{l}(n)\sigma^{(n)} . \tag{8}$$

In the next section we will use only the transverse part ($\mu = \nu = x$ and y) of sum rule (7), which is given by

$$\int \vec{p}_{2\perp} \cdot \vec{p}_{1\perp} \frac{d^2\sigma^{(n)}}{dp_1 dp_2} dp_1 dp_2 = - \int p_{2\perp}^2 \frac{d\sigma^{(n)}}{dp_2} dp_2 - \int \vec{k}_\perp \cdot \vec{p}_{2\perp} \frac{d\sigma^{(n)}}{dp_2 dk} dp_2 dk . \tag{9}$$

The sum rule (7) clearly provides a constraint of energy-momentum conservation for models of two-particle differential cross sections.

III. TRANSVERSE-MOMENTUM FLUCTUATIONS

For an n-prong event we consider the quantity

$$\vec{\phi}(\bar{y}) = \sum_{i=1}^{n} \vec{p}_{\perp i}\,\theta(\bar{y} - y_i)\,, \tag{10}$$

which involves the sum of transverse momenta of all (charged) particles to the left of $\bar{y}$ on the rapidity plot. For an exclusive n-particle event ϕ would simply be equal to the momentum transfer across $\bar{y}$.

The basic function related to transverse-momentum fluctuations, which we will consider in the following, and which measures the correlation between different $\vec{\phi}$'s, is[6]

$$D_n^2(\bar{y}_2, \bar{y}_1) = \langle \vec{\phi}(\bar{y}_2)\vec{\phi}(\bar{y}_1)\rangle - \langle \vec{\phi}(\bar{y}_2)\rangle\langle \vec{\phi}(\bar{y}_1)\rangle\,, \tag{11}$$

where the average is over all events.

It is straightforward to evaluate $D_n^2(\bar{y}_2, \bar{y}_1)$ once it is realized that the probability $P(\vec{\phi}(\bar{y}))$ for a given $\vec{\phi}(\bar{y})$ is simply

$$P(\vec{\phi}(\bar{y})) = \frac{1}{\sigma^{(n)}} \int \frac{d\sigma^{(n)}}{dp_1 \cdots dp_n}\,\delta^2\left(\vec{\phi}(\bar{y}) - \sum_{i=1}^{n} \vec{p}_{\perp i}\,\theta(\bar{y} - \bar{y}_i)\right) dp_1 \cdots dp_n\,.$$

One then finds that $\langle \vec{\phi}(\bar{y})\rangle = 0$ and, taking $\bar{y}_2 > \bar{y}_1$, that

$$D_n^2(\bar{y}_2, \bar{y}_1) = \int_{-\infty}^{\bar{y}_1} dy \int p_\perp^2 \frac{d\sigma^{(n)}}{\sigma^{(n)} dy\, d^2p_\perp} d^2p_\perp + \int_{-\infty}^{\bar{y}_1} dy_1 \int_{-\infty}^{\bar{y}_2} dy_2 \int \vec{p}_{\perp 1}\cdot\vec{p}_{\perp 2} \frac{d\sigma^{(n)}}{\sigma^{(n)} dy_1 dy_2 d^2p_{\perp 1} d^2p_{\perp 2}} d^2p_{\perp 1} d^2p_{\perp 2}\,. \tag{12}$$

where we have considered a rapidity interval from $-\infty$ to $+\infty$. The expression (12) can be rewritten as

$$D_n^2(\bar{y}_2, \bar{y}_1) = \int_{-\infty}^{\bar{y}_1} dy \int p_\perp^2 \frac{d\sigma^{(n)}}{\sigma^{(n)} dy\, d^2p_\perp} d^2p_\perp + \int_{-\infty}^{\bar{y}_1} dy_1 \int_{-\infty}^{\infty} dy_2 \int \vec{p}_{\perp 1}\cdot\vec{p}_{\perp 2} \frac{d\sigma^{(n)}}{\sigma^{(n)} dy_1 dy_2 d^2p_{\perp 1} d^2p_{\perp 2}} d^2p_{\perp 1} d^2p_{\perp 2}$$

$$- \int_{-\infty}^{\bar{y}_1} dy_1 \int_{\bar{y}_2}^{\infty} dy_2 \int \vec{p}_{\perp 1}\cdot\vec{p}_{\perp 2} \frac{d\sigma^{(n)}}{\sigma^{(n)} dy_1 dy_2 d^2p_{\perp 1} d^2p_{\perp 2}} d^2p_{\perp 1} d^2p_{\perp 2}\,.$$

With the help of sum rule (9), the two first terms on the right-hand side of this expression can now be expressed in terms of the cross section for producing one charged and one neutral particle in an n-prong event. This is, however, only possible if we consider that the single-particle distribution in (9) does not depend on y and that the two-particle distributions depend on their rapidity difference only. For the latter case, one can equivalently talk about correlation functions because in sum rule (9) any two-particle distribution can be replaced by the corresponding two-particle correlation function, since the azimuthal dependence of single-particle cross sections is trivial. Using sum rule (9) then, with the above restrictions, we arrive at the final result:

$$D_n^2(\bar{y}_2, \bar{y}_1) = -\int_{-\infty}^{\bar{y}_1} dy_1 \int_{\bar{y}_2}^{\infty} dy_2 \vec{p}_{\perp 1}\cdot\vec{p}_{\perp 2} \frac{d\sigma^{(n)}}{\sigma^{(n)} dy_1 dy_2 d^2p_{\perp 1} d^2p_{\perp 2}} d^2p_{\perp 1} d^2p_{\perp 2}$$

$$- \int_{-\infty}^{\bar{y}_1} dy \int_{-\infty}^{\infty} dy_0 \vec{k}\cdot\vec{p}_\perp \frac{d\sigma^{(n)}}{\sigma^{(n)} dy\, dy_0 d^2k\, d^2p_\perp} d^2k\, d^2p_{\perp 1}\,, \tag{13}$$

which we write as

$$D_n^2(\bar{y}_2, \bar{y}_1) = D_C(\bar{y}_2, \bar{y}_1) + D_0(\bar{y}_2, \bar{y}_1)\,, \tag{14}$$

where D_C involves the two-charged-particle cross section and D_0 the charged-neutral one. Compared with the exclusive case[7] the new ingredient is the presence of the second term on the right-hand side of (13), with the neutral-charged differential cross section normalized as in (8).

The physical meaning of the first term on the right-hand side of the expression for $D_n^2(\bar{y}_2, \bar{y}_1)$ is clear. It represents the probability of finding particle 1 to the left of $\bar{y}_1$ with transverse momentum $\vec{p}_{\perp 1}$ and particle 2 to the right of $\bar{y}_2$ with momentum $\vec{p}_{\perp 2}$ and measures the average value of $\vec{p}_{\perp 1}\cdot\vec{p}_{\perp 2}$ in all such cases. This term therefore contains important physical information as to the correlation in rapidity space of the transverse momenta of two charged particles. Before dis-

cussing $D_C(\bar{y}_2, \bar{y}_1)$ in more detail, one must ask whether there is any model-independent way to determine that term in the expression of $D_n^2(\bar{y}_2, \bar{y}_1)$ [Eq. (13)] which involves a neutral particle, namely, $D_0(\bar{y}_2, \bar{y}_1)$. The answer is yes. We first remark that D_0, which does not depend on $\bar{y}_2$, does not depend on $\bar{y}_1$ either, because we are limited to the case where the two-particle distributions depend on the difference only of particle rapidities. To obtain D_0 then, it is sufficient to consider transverse-momentum fluctuations over the whole rapidity space Y, i.e., fluctuations of the quantity $\vec{\phi} = \sum_{i=1}^{n} \vec{p}_{\perp i}$. This fluctuation measures the average value of $\vec{k} \cdot \vec{p}_\perp$, where $\vec{k}$ is the transverse momentum of a neutral particle in an n-prong event. One therefore gains information about a property of neutral- and charged-particle correlation *without* ever measuring the transverse momentum of the neutral particle. In the rapidity region where D_0 is independent of $\bar{y}_1$, it is expected to be given by this last measurement, equal in fact to $\frac{1}{2}$ of its value, since the lower bound $-\infty$ on the integral stands for $-Y/2$. It is thus possible to isolate the D_0 term in (13). We are therefore justified to discuss separately the term

$$D_C(\bar{y}_2, \bar{y}_1) = -\int_{-\infty}^{\bar{y}_1} dy_1 \int_{\bar{y}_2}^{\infty} dy_2 \vec{p}_{\perp 1} \cdot \vec{p}_{\perp 2} \frac{d\sigma^{(n)}}{\sigma^{(n)} dy_1 dy_2 d^2p_{\perp 1} d^2p_{\perp 2}} d^2p_{\perp 1} d^2p_{\perp 2} . \tag{15}$$

As stated before, our considerations are limited to the case where the differential cross section (or, equivalently, in this case, the correlation function) in (15) depends on the rapidity difference only. If such is the case, then $D_C(\bar{y}_2, \bar{y}_1)$ itself depends only on the length $\Delta = \bar{y}_2 - \bar{y}_1$. This fact is assumed to hold in the overlap-function calculations,[1] which, of course, apply to *exclusive* n-particle channels. The distance over which $D_C(\Delta)$ will fall off characterizes the correlation between the two charged particles. Expressed in terms of clusters, the point is whether the correlation becomes small once Δ exceeds the distance of 1 to 1.5 units of rapidity over which the cluster effects seem to manifest themselves.[9] Moreover, *if* the average transverse momentum is independent of energy, it follows from considering the normalizations of two-particle cross sections that, in the region where only the rapidity difference matters, $D_C(\Delta)$ is expected at fixed n to decrease as 1/log$s$, while the energy dependence of D_0 will be entirely contained in $\bar{l}(n)$, the average number of neutrals for a given prong number (see also Ref. 3).

As we have mentioned, it is interesting that much can be learned about the *neutral*-charged cross section in an n-prong event from measuring *charged*-particle fluctuations only. In those cases where independent measurements of the charged-neutral two-particle cross section exist, they can of course be used either directly in expression (13) or as a check for the results derived from expression (13) of the fluctuation. For instance, in the case where $D_C(\Delta)$ goes to zero sufficiently fast with increasing Δ, i.e., when long-range transverse-momentum effects are small, then the fluctuation $D_n^2(\Delta)$ gives precisely the value of $D_0(\bar{y}_1)$, and one can compare it with any direct knowledge of the two-particle charged-neutral correlation.

At this point, we are unable to suggest a series of "significant" experiments to be performed. As emphasized in the Introduction, our aim has been to exhibit clearly the formalism with which transverse-momentum fluctuations can be expressed in semi-inclusive reactions. It is a question of facing the fact that not *all* neutral particles can be detected. The formalism has some usual limitations (infinite rapidity interval, rapidity difference dependence only), but it is remarkable that the term contributing to the fluctuation which involves the neutral particle can be determined, in principle, by experimental measurement of the charged fluctuations only. It is, of course, our belief that semi-inclusive reactions can provide useful information on the over-all transverse-momentum structure of multiparticle events, which will help clarify the theoretical issue of locality. Whether such a belief is justified, only experiment can tell.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge interesting conversations with C. Quigg, and to thank P. Hoyer for very useful criticism and comments on the manuscript.

*Work supported in part by National Science Foundation under Grant No. GP-32995X.

†Permanent address: Service de Physique Théorique, C.E.N. Saclay, 91190 Gif-sur-Yvette, France.

[1]A. Krzywicki and D. Weingarten, Phys. Lett. B 50, 265 (1974); Nucl. Phys. B86, 296 (1975); Orsay Report No. LPTPE 75/10 (unpublished); D. Weingarten, Phys. Rev. D 11, 1924 (1975). See also P. Grassberger and H. I. Miettinen, Nucl. Phys. B92, 309 (1975).

[2]J. Kubar-Andre, M. Le Bellac, and J. L. Meunier, Nucl. Phys. B93, 138 (1975).

[3]This question has also been studied by C. Quigg within the framework of a cluster "ω" model [Phys. Rev. D 12, 834 (1975)].

[4]M. Pratap *et al.*, Phys. Rev. Lett. 33, 797 (1974); K. Eggert *et al.*, Nucl. Phys. B86, 201 (1975); B. Y. Oh *et al.* (Michigan State, ANL, Fermilab, Iowa State and Ames Laboratory, and University of Maryland Collaboration), Phys. Lett. 56B, 400 (1975).

[5]C. E. DeTar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D 4, 906 (1971); L. S. Brown, *ibid.* 5, 748 (1972).

[6]This function, in the exclusive case, plays an essential part in the work of Ref. 1: Its integral over the rapidity difference $|\bar{y}_2 - \bar{y}_1|$ is directly related to the lower bound on the Pomeron slope.

[7]For exclusive experiments the expression for $D_n^2(\bar{y}_2, \bar{y}_1)$, when $\bar{y}_2 = \bar{y}_1$, has been used previously (see Ref. 8) as applied to the reactions $pp \to pp + (n$ charged pions) at 24 GeV.

[8]A. Białas *et al.*, Nucl. Phys. B86, 365 (1975).

[9]One has to consider a situation where Δ can take large enough values without exceeding, say, half of the available rapidity interval. Such a reservation is necessary because $D_C(\Delta)$ goes to zero when $\Delta \to \infty$.